Section A (20 points)

1. C	5. A	9. A	13. D	17. E
2. E	6. B	10. E	14. A	18. B
3. C	7. E	11. C	15. B	19. B
4. D	8. B	12. E	16. D	20. D

Section B (112 points)

21. (2 points) 4, 3, 2, 1, 5 (all or nothing)

22. (2 points) 3, 5, 4, 1, 2 (all or nothing)

23. (a) Saturn
    (b) Water ice
    (c) No
    (d) Image 21

24. (a) Jupiter
    (b) Juno
    (c) Image 19

25. (a) Image 6
    (b) No
    (c) Mathematical calculations of its predicted position due to observed perturbations in the orbit of Uranus

26. (a) Ceres
    (b) It started to form but didn't finish, leaving it as a frozen "planetary embryo".
    (c) Occator Crater
    (d) After brine (salty water) eruptions, the water sublimates and leaves behind salt deposits

27. (a) MESSENGER
    (b) It let it crash into the surface of Mercury
    (c) BepiColombo

28. (a) Pluto
    (b) New Horizons
    (c) Image 20

29. (a) Image 13

   (b) Solar heating and cosmic rays can alter the surface of a comet. If we can study the comet far enough *below* the surface, we can probe cometary material that is essentially unchanged since the formation of the Solar System.

30. (a) The Moon
    (b) The same side of the Moon faces Earth at all times (approximately)
    (c) Dark plains on the formed billions of years ago by ancient volcanic lava flows filling large impact basins

31. (a) Uranus
    (b) Hubble Space Telescope
    (c) A moon of Uranus (white) and its shadow (black)

32. (a) 25143 Itokawa
    (b) Hayabusa

33. (a) Image 12
    (b) Triton
    (c) Image 18
    (d) Image 6

34. (a) Image 15
    (b) Infrared
    (c) Yes, it is younger than Earth. Beta Pictoris is surrounded by a massive, chaotic disk of gas and dust, which is still leftover from the system's formation since it happened so recently. In our own Solar System, we do not see this.

35. (a) Radial velocity
    (b) It formed further out and later migrated inwards

36. (a) Uranus
    (b) Hubble Space Telescope
    (c) A cloud

37. (a) Image 4
    (b) Compression
    (c) As Mercury cools, it shrinks. The material has nowhere to go, so it pushes into itself, leading to compressive forces

38. (a) When an object impacts the Moon, it "kicks up" a lot of material in the process, throwing it upward and outward to settle around the newly formed crater
    (b) It is "fresh" material that has not yet been darkened by millions of years of exposure to solar radiation and micromete-

orite bombardment, which makes material on the Moon darker

(c) Decreases

(d) The meteorite came in at an angle (not just straight down and perpendicular to the surface). The side that the meteorite came from will have less ejecta.

39. (a) 486958 Arrokoth

(b) New Horizons

(c) C is on the large lobe, which is bigger. With equal density, the larger lobe has a larger surface gravity scale (more mass nearby). Also, the small lobe's gravity can add to the net gravity on parts of the large lobe.

(d) B is in the neck, so the gravitational pulls from the two lobes point in different directions and partially cancel, so the net effective gravity magnitude can be very small there.

(e) Decreasing rotation rate means smaller centrifugal acceleration and less reduction of gravity. So, the effective surface gravity at A would increase.

Section C (68 points)

40. (a) (3 points) Mercury: 1; Neptune: 2; Jupiter: 3 (+1 each for a total of +3)

(b) (2 points) No. (+1) Measuring M and R uniquely determines only the planet's *mean density*. Many different internal compositions/structures can share the same (M, R) (e.g., different mixtures of iron, silicates, water/ices, and a H/He envelope; different core sizes; different temperature/entropy states). (+1) Additional observations (atmospheric spectra, gravitational moments, etc.) are needed to better constrain composition.

(c) (3 points) For a constant-density, homogeneous sphere,

$$M = \rho V = \rho \left(\frac{4}{3} \pi R^3 \right)$$

Solving for R gives

$$R = \left(\frac{3M}{4\pi\rho} \right)^{1/3} \propto M^{1/3}.$$

Therefore, in $R \propto M^x$, we have

$$\boxed{x = \frac{1}{3}}$$

(d) (4 points) Start with

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \propto \frac{M}{R^3}$$

If $R \propto M^x$, then $R^3 \propto M^{3x}$, so

$$\rho \propto \frac{M}{M^{3x}} = M^{\,1-3x}$$

Thus, if $\rho \propto M^y$, the exponents satisfy

$$\boxed{y = 1 - 3x}$$

(e) (4 points) Region 2 corresponds to planets that have grown massive enough (via core accretion) that they can hold onto a significant amount of gas. (+2) Adding a comparatively small amount of gas can increase the radius a lot because it occupies a large volume, while the added mass is not as large relative to the volume increase; the accreted gas is much less dense than the rocky core, so the overall density goes down. (+2) The *bulk* density decreases because R^3 grows faster than M.

(f) (4 points) Region 3 corresponds to gas giants where additional mass mainly causes *gravitational compression* rather than expansion. (+2) As M increases, the planet's self-gravity compresses the gas envelope. This can keep R nearly constant or even make it decrease slightly, while M continues to rise, so the bulk density increases because M rises while R changes little (or falls). (+2)

41. (a) (2 points) It is young (on geological timescales).

(b) (4 points) N_2, CO, and CH_4. (+1 each for a total of +3) Of these, N_2 is the most prevalent on Sputnik Planitia. (+1)

(c) (2 points) Ice is warmed by heat from the interior, causing it to well up

(d) (4 points) It is pretty flat. (+2) This implies that the viscosity of the ice is relatively low (the ice was able to change shape on a short enough timescale). (+2)

(e) (4 points) Local depressions receive reflected light energy from surrounding slopes, enhancing sublimation rates on depression floors and making the pits deeper. (+2) However, this is opposed by the movement of the ice "filling in" the gaps as it goes down the slope. (+2)

(f) (4 points) The pits will continue to evolve until the timescales for sublimation and "slope diffusion" are comparable. (+2) When the slope diffusivity increases, it acts faster, so sublimation can only compete with slope diffusion for large pits. (+2) When the slope diffusivity is very small, sublimation would lead to the formation of structures like penitentes on Earth.

(g) (2 points) Answers will vary. One possible answer is that different parts of the ice sublimate at different rates. In the paper, for example, the authors imposed that slopes facing to upper left sublimate 3 times more rapidly than comparable slopes facing in other directions, leading to the structure we saw in panel (d) on the exam.

42. (a) (2 points) From the diagram, θ is the angle between the orbital plane (hypotenuse) and the line of sight. The component of the radius perpendicular to the line of sight, $d_\perp$, is opposite to the angle. The sine of θ is opposite over hypotenuse; i.e., $\sin\theta = d_\perp/a$. So $\boxed{d_\perp = a\sin\theta}$.

(b) (4 points) A transit occurs when the sky-projected separation between the centers of the star and planet is no more than the sum of their radii; i.e., $d_\perp \le R_s + R_p$. The maximum value of θ will occur when it is equal to $R_p + R_s$. From the previous part, we know $d_\perp = a\sin\theta$. So,

$$a|\sin\theta_{\max}| = R_s + R_p \quad\Longrightarrow\quad \boxed{|\sin\theta_{\max}| = \frac{R_s + R_p}{a}}$$

(c) (4 points) All viewing directions that produce a transit fall within $-\theta_{\max} \le \theta \le \theta_{\max}$. On a unit sphere, a point at angle θ above the equator has height $z = \sin\theta$. The band spans heights:

$$z_1 = \sin(-\theta_{\max}) = -\sin\theta_{\max}, \qquad z_2 = \sin(\theta_{\max}) = \sin\theta_{\max}$$

The belt (band) area is then

$$A_{\text{band}} = 2\pi(z_2 - z_1) = 2\pi\big(\sin\theta_{\max} - (-\sin\theta_{\max})\big) = \boxed{4\pi\sin\theta_{\max}}$$

(d) (4 points) The transit probability is the fraction of the unit sphere covered by the band:

$$P_{\text{tr}} = \frac{A_{\text{band}}}{4\pi} = \frac{4\pi\sin\theta_{\max}}{4\pi} = \sin\theta_{\max}$$

From the transit condition at the boundary of the band,

$$a\sin\theta_{\max} = R_s + R_p \quad\Longrightarrow\quad \sin\theta_{\max} = \frac{R_s + R_p}{a}$$

Therefore,

$$\boxed{P_{\text{tr}} = \frac{R_s + R_p}{a}}$$

(e) (4 points) For Earth–Sun,

$$\frac{R_p}{R_s} \sim 10^{-2}.$$

Using R_s/a instead of $(R_s + R_p)/a$ makes a *fractional* error of

$$\frac{(R_s + R_p)/a - R_s/a}{(R_s + R_p)/a} = \frac{R_p}{R_s + R_p} \approx \frac{R_p}{R_s} \sim \boxed{10^{-2}}$$

(f) (4 points) These expressions give the probability that a transit is *geometrically possible* (i.e., the orbit is aligned closely enough that the planet crosses the star on the sky). Detecting a transit also requires that the signal be measurable. Other things we must consider include:

- The dip in brightness must be large enough compared to noise
- Enough transits must be observed to be confident it is real
- Real observations have limitations (instrument sensitivity, gaps in data, stellar variability, etc.)

So geometry tells you whether a transit *could* happen from a random viewing direction, while detectability tells you whether your telescope and observing strategy can actually *measure* it.

(g) (4 points) The transit method is best suited to planets that (i) have a high geometric transit probability and (ii) produce a large, repeatable brightness dip:

- **Close-in planets** (small a) have higher transit probability (+1) and shorter periods mean transit occur more often (+1) during a single observing campaign.
- **Large planets** (large R_p) have deeper transits. (+1)
- **Around smaller stars** (small R_s), the same-sized planet will have a deeper transit. (+1)

43. Tiebreaker — answers will vary.