Section A (20 points)

1. C	5. A	9. C	13. E	17. A
2. E	6. C	10. D	14. A	18. E
3. C	7. A	11. A	15. B	19. B
4. D	8. D	12. B	16. C	20. E

Section B (80 points)

21. (2 points) 5, 1, 2, 4, 3 (all or nothing)

22. (2 points) 8, 12, 5, 2, 1 (all or nothing)

23. (a) Jupiter
    (b) A large storm

24. (a) Saturn
    (b) Water ice
    (c) Cassini

25. (a) Neptune
    (b) Voyager 2
    (c) Mathematical predictions

26. (a) Uranus
    (b) Hubble Space Telescope

27. (a) Mercury
    (b) Clusters of rimless depressions with flat floors and halos of bright material surrounding them (from Wikipedia)
    (c) Image 11
    (d) Young. The longer Eminescu has been around, the more time there has been for stuff to fall on Eminescu and create more craters. Since we do not see many craters on top of Eminescu, Eminescu itself must be relatively young.

28. (a) Tempel 1
    (b) Image 16

29. (a) New Horizons
    (b) Pluto is not massive/gravitationally dominant enough to "clear its neighborhood"

30. (a) Pluto

    (b) Sputnik Planitia

    (c) Nitrogen

31. (a) Image 15

    (b) A type of complex organic compounds formed when simple molecules like methane and nitrogen are irradiated by energetic particles, such as cosmic rays or ultraviolet light

32. (a) Ceres

    (b) After brine (salty water) eruptions, the water sublimates and leaves behind salt deposits

    (c) Image 10

    (d) Dawn

33. (a) 25143 Itokawa

    (b) It may be formed from two distinct entities that have gravitated toward each other and stuck together

34. (a) Beta Pictoris

    (b) Large quantities of dust and gas (including carbon monoxide)

35. (a) Earth's Moon

    (b) Low, curvy ridges created after lava cooled and solidified

36. (a) Triton

    (b) Image 6

    (c) Voyager 2

37. (a) HD 209458

    (b) Radial velocity

Section C (54 points)

38. (a) (4 points) Thermal requirements: Mercury is close to the Sun, so the spacecraft must be able to withstand high temperatures (and high temperature differences)

    Propulsion: Going towards Mercury liberates a lot of gravitational potential energy and speeds up the spacecraft. So, the spacecraft must be able to slow down significantly so it can orbit Mercury.

    (b) (4 points) Weakens. The more "volatile" a substance, the easier it is to evaporate off (by definition). If the crust/mantle were evaporated off due to high temperatures, then volatile materials like potassium would not be as prevalent on Mercury.

    (c) (4 points) After the collision, there will be a lot of debris on orbits that intersect with Mercury's orbit. Numerical simulations (e.g., Carter *et al.*, 2015) suggest that much of the debris would be reaccreted quickly and the mass will not change that much. Possible workarounds: Answers include, but are not limited to: (i) if the debris particles are small enough, Poynting-Robertson drag can pull them into the Sun and (ii) proto-Mercury collided with a larger object, most of the debris accreted onto the larger object, and the post-collision proto-Mercury became Mercury.

39. (a) (2 points) An object's gravity pulls the trajectories of smaller moving objects towards it, effectively increasing the object's "collision cross-section".

    (b) (2 points) The mass within the feeding zone will be the product of the area of the feeding zone and the surface mass density of the feedzing zone. We are given that the area is $2\pi d \times (2\Delta d) = 4\pi d\Delta d$ and that the surface mass density is Σ. So, the total mass is:

$$M = 2\pi d \times (2\Delta d) \times \Sigma = 4\pi \Sigma d\Delta d$$

(c) (4 points) Setting $M_\text{p} = M_\text{iso} = M_\text{feed}$ and substituting our expression for Δd from the previous part gives:

$$M_\text{iso} = 4\pi\Sigma d \cdot 2\sqrt{3} \cdot d \left(\frac{M_\text{iso}}{3M_\text{s}}\right)^{1/3}$$

Bringing all M_iso to one side and solving for M_iso gives:

$$\begin{aligned} M_\text{iso}^{2/3} &= 4\pi\Sigma d \cdot 2\sqrt{3} \cdot d \left(\frac{1}{3M_\text{s}}\right)^{1/3} \\ M_\text{iso} &= \frac{8}{\sqrt{3}}\pi^{3/2}\left(2\sqrt{3}\right)^{3/2} M_\text{s}^{-1/2}\Sigma^{3/2}d^3 \\ &= \boxed{C \cdot M_\text{s}^{-1/2}\Sigma^{3/2}d^3} \end{aligned}$$

where all of the dimensionless constants are lumped into C for clarity.

(d) (4 points) The frost line is the minimum distance from the central (proto)star where the temperature is low enough for volatile compounds (such as water) to condense into solid grains. Since gaseous substances are now cool enough to be solid, the surface density of solids jumps up, which makes core growth easier. (In other words, it is cold enough for there to be more solid material to build a planet out of.)

40. (a) (3 points) The flux will be the luminosity of the star divided by the surface area of a sphere of radius d:

$$\boxed{F = \frac{L_\text{s}}{4\pi d^2}}$$

(b) (3 points) The luminosity of the planet is the amount of light it reflects. The power of light incident on the planet is the product of the planet's cross-sectional surface area and the incident flux. An object of albedo α reflects that proportion of its incoming light, so:

$$\begin{aligned} L_\text{p} &= F \cdot \pi R_\text{p}^2 \cdot \alpha \\ &= \frac{L_\text{s}}{4\pi d^2} \cdot \pi R_\text{p}^2 \cdot \alpha = \boxed{\frac{L_\text{s} R_\text{p}^2 \alpha}{4d^2}} \end{aligned}$$

(c) (4 points) The ratio $L_\mathrm{p}/L_\mathrm{s}$ is:

$$\begin{aligned} L_\mathrm{p}/L_\mathrm{s} &= \frac{L_\mathrm{s} R_\mathrm{p}^2 \alpha / 4d^2}{L_\mathrm{s}} \\ &= \frac{R_\mathrm{p}^2 \alpha}{4d^2} \end{aligned}$$

A typical albedo for a gas planet in the Solar System is around 0.5. Even if we didn't know that, we do know albedo has to be between 0 and 1, so unless we chose a *really* small albedo (which would be pretty unrealistic for a planet like Jupiter) our order-of-magnitude estimate would not change much. Plugging in concrete numbers, we get:

$$\begin{aligned} L_\mathrm{p}/L_\mathrm{s} &= \frac{R_\mathrm{p}^2 \alpha}{4d^2} \\ &= \frac{\left(7 \times 10^7\right)^2 \cdot 0.5}{4\left(8 \times 10^{11}\right)^2} \\ &\approx \frac{10^{14}}{10^{23}} = \boxed{10^{-9}} \end{aligned}$$

Answers ± 1 order of magnitude (i.e., answers that are 10^{-10}, 10^{-9}, or 10^{-8}) are all fine.

(d) (2 points) At $\theta = 0$ arcseconds, the flux is about 10^{-1}. At $\theta = 1$ arcsecond, the flux is about 10^{-5}. So, the star's intensity becomes 10^{-4} of relative to the center. (This is the same as saying it does down by a factor of 10^4.)

(e) (2 points) From the problem statement, we know that the noise is proportional to the square root of the signal. If the signal becomes 10^{-4} of what it used to be (as we calculated in the previous question), then the noise will become $\sqrt{10^{-4}} = 10^{-2}$ of what it used to be. In other words, the noise will go down by a factor of 10^2.

(f) (4 points) Our answer to part (c) tells us that the signal of the planet relative to the center of the star is on the order of 10^{-9}. Our answer to part (e) tells us that the noise from the star at the planet's location, relative to the signal at center of the star, is on the order of 10^{-2}. In order for us to find the planet, we'd want $S_\mathrm{p} \gtrsim N_\mathrm{s}(\theta)$, but here, we're many orders of magnitude short.

Essentially, this is like trying to find a firefly in front of a lighthouse, but the lighthouse's brightness fluctuates on a scale 10^7 times larger than the firefly is bright. Note: this is with data from the Hubble Space Telescope, which is in space. If we were looking with a telescope based on Earth, then we'd have to factor in additional noise from the atmosphere, which makes things much worse.

(g) (6 points) (i) Planets emit a significant amount of light in the infrared. Furthermore, the ratio of emitted blackbody radiation per unit area for the planet vs. star is much higher in the infrared than visible. All in all, this makes the planet brighter in infrared than in visible light relative to the star. (ii) A coronagraph is a device that blocks or destructively interferes light from the star in the center, so that objects further from the center do not get covered up. It's similar to using your hand to block the Sun so you can look at something else more clearly. Adaptive optics refers to a setup where the mirror in a telescope deforms so that distortions due to Earth's atmosphere are corrected in real time.

(h) (6 points) Young: young planets are hotter than old planets and therefore emit more light than old planets. (+2) Hot: hot planets emit more light than cool planets. (+2) Far from their parent stars: the planet's light won't be drowned out as much and it is easier to block out the light from the parent star. (+2)

41. Tiebreaker — answers will vary.