Section A (20 points)

1. C
2. C
3. B
4. A
5. B
6. A
7. A
8. C
9. E
10. A
11. A
12. B
13. D
14. E
15. B
16. A
17. A
18. C
19. C
20. D

Section B (125 points)

21. (4 points) 5, 2, 3, 1, 4 (all or nothing)

22. (a) Pluto

    (b) New Horizons

    (c) In short, it would have required significantly more fuel, making the spacecraft too heavy and expensive to launch, or it would have taken much longer to get to Pluto. I encourage you to read these posts ([1], [2], [3]) on the Space Exploration StackExchange for more discussion.

    (d) Tombuagh Regio

23. (a) Saturn

    (b) Many particles of ice, dust, and rock, ranging in size from microscopic grains to large boulders

    (c) Ammonia crystals in its upper atmosphere

24. (a) Image 6

    (b) Tempel 1

    (c) Deep Impact

25. (a) Jupiter

    (b) A big storm (the biggest in the Solar System, in fact) on Jupiter

    (c) Image 15

    (d) Hubble Space Telescope

    (e) UV

26. (a) 486958 Arrokoth

    (b) New Horizons

    (c) It is the result of a gentle, low-speed merger of two separate objects

27. (a) 25143 Itokawa
    (b) JAXA/Japan
    (c) Image 13
28. (a) Images 5 and 17
    (b) MESSENGER
    (c) Rocky/metallic
29. (a) Ceres
    (b) Ahuna Mons
    (c) The mountain is younger than the surrounding terrain
    (d) A tholus is a "small domical mountain or hill"
    (e) Accept answers between 2.5 and 4 km
    (f) Kwanzaa Tholus is most likely older. Earlier in Ceres' history, it was likely similar in size to Ahuna Mons, but because ice isn't very strong, it's since degraded and become shorter.
30. (a) Image 3
    (b) Canteloupe terrain
    (c) Voyager 2
    (d) Image 16
    (e) Answers include: Triton has a retrograde orbit (it orbits in the direction opposite to Neptune's rotation) and Triton's composition closely matches that of other objects in the Kuiper Belt.
31. (a) Image 11
    (b) ALMA
    (c) Radio
    (d) When viewed in visible light, HL Tauri's central star and protoplanetary disk would be shrouded in an opaque envelope of dust and gas.
    (e) As a planet forms and grows, its gravity attracts the surrounding material, accumulating it onto itself. This sweeping action creates a noticeable void, or gap, around the orbit in the disk where the planet is located
32. (a) Uranus
    (b) Hubble Space Telescope
    (c) A false color image assigns visible light colors (red, green, blue) to non-visible wavelengths of the electromagnetic spectrum (like infrared) or to different parts of the visible spectrum, which differs from a true-color image that shows colors as they appear to the human eye.

(d) Clouds

(e) Moons of Uranus

33. (a) 51 Pegasi b

(b) x axis: phase of orbit; y axis: radial velocity of 51 Pegasi (not 51 Pegasi b)

34. (a) Left: microwave; middle: visible; right: infrared

(b) Juno

(c) Left: Microwave Radiometer (MWR); middle: JunoCam; right: Jovian Infrared Auroral Mapper (JIRAM)

(d) Visible light (shallowest), then infrared, then microwave

(e) The central cyclone either does not extend as deep as the other storms, or it has a very different subsurface structure.

35. (a) The Moon

(b) Lunar Reconnaissance Orbiter

(c) A crater. From the elevation bar, we can see that after we cross the rim, the elevation goes down as we approach the center.

(d) Tycho Crater

(e) The surface feature in Image 19 (Tycho Crater) is larger. Small craters are typically small and bowl-shaped with smooth walls, while large craters feature more complexity and intricate structures like central peaks. The crater in Image 18 has smooth walls and is clearly shaped like a bowl. From the scale bar, we can see that it is only a few kilometers across. The create in Image 19 has all sorts of complicated features, including the peak in the center.

36. (a) HD 209458 b

(b) Artist illustration. We do not have the technology to take an image with that level of detail of HD 209458 b. We discovered HD 209458 b by looking at small dips in the brightness of the star it orbits.

37. (a) Beta Pictoris

(b) A young star system with significant circumstellar material and at least two planets.

Section C (70 points)

38. (a) Let f_V be the fraction of the object's *volume* that is ice. The mean density is the volume–weighted average:

$$\bar{\rho} = \rho_{\text{ice}} f_V + \rho_{\text{rock}}(1 - f_V) = 0.9\, f_V + 3.6\,(1 - f_V).$$

Setting $\bar{\rho} = 1.8$ and solving gives

$$1.8 = 3.6 - 2.7 f_V \;\Rightarrow\; f_V = \frac{1.8}{2.7} = \boxed{\frac{2}{3}}.$$

(b) The ice mass fraction is the ice mass divided by the total mass:

$$f_M = \frac{\rho_{\text{ice}} V_{\text{ice}}}{\rho_{\text{ice}} V_{\text{ice}} + \rho_{\text{rock}} V_{\text{rock}}} = \frac{\rho_{\text{ice}} f_V}{\bar{\rho}} = \frac{0.9 \times (2/3)}{1.8} = \boxed{\frac{1}{3}}.$$

(c) The volume fraction of rock is $1 - f_V = 1/3$. If the rock forms a spherical core of radius R_c inside a sphere of total radius R, then:

$$\frac{1}{3} = \frac{\cancel{\frac{4}{3}\pi} R_{\text{core}}^3}{\cancel{\frac{4}{3}\pi} R_{\text{total}}^3} \Rightarrow \boxed{\frac{R_{\text{core}}}{R_{\text{total}}} = \left(\frac{1}{3}\right)^{1/3}}$$

(d) When the total mass is fixed, changing the distribution of mass within the object by moving mass outwards increases moment of inertia (I). When the object is differentiated, the highest density material is at the center, concentrating mass at the center. However, a uniform mixture puts relatively *more* mass at larger radii than a dense rocky core with an icy shell, so $\boxed{I_{\text{uniform}} > I_{\text{differentiated}}}$.

(e) If the radius is the same and the mass is now greater than it used to be, the density is greater than we originally thought. That means the denser material (rock) must make up a larger fraction of the object than before. <u>So, our answers to parts (a) and (b) would go down, while our answer to part (c) would go up.</u>

(f) Yes. Larger bodies have stronger gravity and higher internal heating, promoting melting and separation of materials. Thus the assumption of complete differentiation becomes **better** for larger objects.

39. (a) (4 points) An RTG (radioisotope thermoelectric generator) converts the heat released by the natural radioactive decay of a fuel (e.g., Pu–238) into electricity using thermocouples (no moving parts). Advantages include: steady power independent of sunlight, long lifetime (decades) and high reliability (few/no moving parts). Disadvantages include: radioisotope availability (part of the reason Juno went with solar panels is due to a plutonium shortage), expensive, handling/safety constraints.

(b) (3 points) Using the inverse-square law and proportions:

$$I_{\text{Nep}} = I_{\oplus}\left(\frac{1\,\text{AU}}{30\,\text{AU}}\right)^2 = 1350\,\frac{\text{W}}{\text{m}^2} \times \frac{1}{900} = \boxed{1.5\ \text{W}\,\text{m}^{-2}}$$

(c) (5 points) Power from panels: $P = \eta\, I_{\text{Nep}}\, A$. Solve for area:

$$A = \frac{P}{\eta I_{\text{Nep}}} = \frac{150\ \text{W}}{0.20 \times 1.5\ \text{W}\,\text{m}^{-2}} = \boxed{500\ \text{m}^2}.$$

Mass (using $1\ \text{kg}\,\text{m}^{-2}$):

$$m_{\text{panels}} = \sigma A = (1\ \text{kg}\,\text{m}^{-2}) \times 500\ \text{m}^2 = \boxed{500\ \text{kg}}.$$

(For scale, a square array of this area would be $\sim \sqrt{500} \approx 22$ m on a side.)

(d) (3 points) Answers will vary. I care more about the reasoning and students knowing the advantages and disadvantages than the final choice they make. Personally, I would choose the RTG. At Neptune the sunlight is so weak that solar power requires an enormous, massive array and strict Sun-pointing, with little margin. For reference, the entire mass of the *New Horizons* probe is about 500 kg, and the mass of the solar panels on *Juno* were only around 340 kg. An RTG provides continuous, reliable power for decades independent of illumination or distance.

40. (a) Free energy consists of a mix of energy and entropy, with the balance between the two mediated by temperature. Mixing is easier at higher temperatures when its entropic benefit of mixing is prioritized more. Since Saturn's interior is cooler than Jupiter, mixing will be less favorable than it is in Jupiter.

(b) Helium droplets are denser than the surrounding hydrogen. As a result, the helium sinks, just like when you put a dense rock in liquid water.

(c) Answers will vary. One example of a possible answer: The mass of Saturn is $100\,M_\oplus = 6 \times 10^{26}$ kg. For simplicity, we'll consider Saturn as having a rocky core surrounded by a gaseous envelope (even though modern *Cassini* data suggests the core is complicated). Assuming that the core is ∼20% of Saturn's mass and that the composition of Saturn's envelope is about the same as the Sun's (∼25% helium), then the mass of helium in Saturn's envelope is $0.25 \times 80 \times 6 \times 10^{24} \approx 10^{26}$ kg. Estimates for the radius of the core vary significantly; let's take it to be half of Saturn's actual radius (i.e., $R_\text{core} \approx 3 \times 10^7$ m). Instead of worrying about uniform density shells or anything like that, we could say the change in gravitational energy is similar to bringing a point mass of mass m_He to a distance of R_core from a point mass of mass m_core. Plugging all of this in:

$$
\begin{aligned}
\Delta U &\approx G m_\text{core} m_\text{He} \left(\frac{1}{R_\text{core}} - \frac{1}{R_\text{middle of envelope}} \right) \\
&\approx \frac{G m_\text{core} m_\text{He}}{R_\text{core}} \\
&\approx \frac{7 \times 10^{-11} \times 20 \times 6 \times 10^{24} \times 10^{26}}{3 \times 10^7} \\
&\approx \boxed{3 \times 10^{34}\ \text{J}}
\end{aligned}
$$

This has many, many, many oversimplifications. But, for an order of magnitude estimation question, we don't worry about the details and we just do our best to put some numbers down. Answers in the range of 10^{33} to 10^{36} would definitely receive full credit, and other answers could too if their assumptions are plausible.

(d) If Saturn emits twice as much energy as it absorbs, then the excess power is equal to the amount of absorbed power. We are told that the radius of Saturn is 6×10^7 meters, so its cross-sectional area is $\pi\left(6 \times 10^7\right)^2 \approx 10^{16}\ \mathrm{m}^2$. The intensity of sunlight at Saturn is:

$$I_{\text{Saturn}} = I_{\oplus}\left(\frac{1\,\mathrm{AU}}{9.5\,\mathrm{AU}}\right)^2 \approx 1350\,\frac{\mathrm{W}}{\mathrm{m}^2} \times \frac{1}{100} \approx 15\,\mathrm{W\,m^{-2}}$$

9.5^2 is a little less than 10^2, which is why I round up to about 15 instead of going with exactly 13.5 (plus, 15 is a "nicer" round number, but it really doesn't matter since we just care about the order of magnitude). Saturn's Bond albedo is 0.4, so:

$$P_{\text{abs}} = P_{\text{excess}} \approx (1 - 0.4) \times 15 \times 10^{16} \approx 10^{17}\ \mathrm{W}$$

The excess energy over the lifetime of the Solar System (which is approximately the lifetime of Saturn) is therefore:

$$E_{\text{excess}} = 3 \times 10^7 \times 4 \times 10^9 \times 10^{17} \approx \boxed{10^{34}\ \mathrm{J}}$$

(e) Our answers to parts (c) and (d) are around the same order of magnitude. Our exercise does not prove anything, but it does suggest that helium rain *could* be a plausible source of the excess energy, possibly along with other factors. If students get different numbers but interpret them in a reasonable way, they would of course get full credit.

41. (a) (2 points) When a cloud of gas and dust collapses under gravity, it often possesses some initial rotation, which gives it angular momentum. The motion in the directions orthogonal to the initial rotation tends to cancel out due to collisions and interactions among particles. This cancellation leads to the formation of a flatter spinning disk, where most of the mass ends up concentrated in this rotating plane. However, since angular momentum is conserved, the disk still spins.

(b) In the core accretion model, a rocky core forms through dust, rocks, etc. colliding and sticking together, gradually getting larger and larger. In some cases, the core becomes so massive that it accretes a significant gaseous atmosphere.

(c) (2 points) The material in the disk orbits around the center according to Kepler's Laws. This means that material that is farther away has a lower angular velocity. This differential rotation means that within a region, the part closer to the center will orbit faster than the part farther from the center. This causes the region to "smear", resulting in the shear force.

(d) (1 point) The force with the **shortest** timescale will dominate.

(e) (3 points) Due to random fluctuations in the disk, sometimes local overdensities will occur. If gravity acts faster than all the other forces, then this overdensity can (might) collapse. However, if the pressure or shear timescales are shorter than that of gravity, the pressure waves and shear forces traveling through the disk will smooth out the overdensity before it is able to collapse (i.e., it becomes harder for a chunk of disk to collapse).

(f) (4 points) In order for the chunk of the disk to collapse, gravity must act on the shortest time scale. In other words, $t_{\rm grav} < t_{\rm shear}$ and $t_{\rm grav} < t_{\rm pr}$. If we solve for Δr, the first condition becomes

$$\Delta r < \frac{G\Sigma}{\Omega^2}$$

Similarly, the second condition becomes

$$\Delta r > \frac{c_{\rm s}^2}{G\Sigma}$$

Putting these two inequalities together, we get

$$\frac{c_{\rm s}^2}{G\Sigma} < \Delta r < \frac{G\Sigma}{\Omega^2} \Rightarrow \frac{c_{\rm s}^2}{G\Sigma} < \frac{G\Sigma}{\Omega^2}$$

The inequality can be rewritten, after taking the square root, as

$$Q \sim \frac{c_{\rm s}\Omega}{G\Sigma} < 1$$

(g) (4 points) As the chunk collapses, it releases gravitational potential energy, so it *also* has to **cool**

quickly enough. Otherwise, it will heat up, which will slow (or stop) the collapse.

42. (5 points) Tiebreaker — answers will vary.