Section A (50 points)

1. [12 pts] Answer true (T) or false (F) for the following questions.
   (a) [1 pt] What separates protostars from pre-main sequence stars is their energy source. **False**
   (b) [1 pt] Some stars have no (discernible) pre-main sequence phase. **True**
   (c) [1 pt] Pre-main sequence stars are less luminous than their main-sequence counterparts. **False**
   (d) [1 pt] Most T Tauri stars are in binary systems. **True**
   (e) [1 pt] Most stars are in binary systems. **True**
   (f) [1 pt] The outer layers of a star are more opaque than the layers closer to the core, which is why many stars have radiative transport in its outer layers. **False**
   (g) [1 pt] Radiative zones are unstable against the formation of convection cells. **False**
   (h) [1 pt] When stars are in hydrostatic equilibrium, they do not move on the H-R diagram. **True**
   (i) [1 pt] Brown dwarfs are more chemically similar to white dwarfs than red dwarfs. **False**
   (j) [1 pt] Due to the presence of high-energy jets, T Tauri stars do not have protoplanetary disks. **False**
   (k) [1 pt] In the planetesimal hypothesis of planet formation, planets are built up in hierarchical collisions. **True**
   (l) [1 pt] The total energy (i.e., the sum of the kinetic energy and potential energy) of a system consisting of one object in an elliptical orbit around another is positive. **False**

2. [8 pts] This question will test your knowledge of a few key terms.
   (a) [1 pt] Do H II regions evolve from molecular clouds, or do molecular clouds evolve from H II regions?
   H II regions evolve from molecular clouds
   (b) [1 pt] What is causing this ionization or recombination?
   Star formation, in particular the formation of young, hot O/B-type stars that have ionizing radiation
   (c) [2 pts] Sort the following terms from smallest to largest: protoplanetary disk, planetesimal, planetary embryo, Earth, the Sun, Jupiter, Sagittarius A*.
   Planetesimal, planetary embryo, Earth, Jupiter, the Sun, Sagittarius A*, protoplanetary disk (0.25 pt per item in correct position)
   (d) [1 pt] The formation of Earth's moon likely resulted from a collision of two of which kind of object? Select from the list in the previous question. **Planetary embryo**
   (e) [1 pt] Which exoplanet detection technique has found the most exoplanets? **Transit method**
   (f) [1 pt] Which exoplanet detection technique has found the second most exoplanets? **Radial velocity method**
   (g) [1 pt] What is the limiting factor for this detection technique? Why has it detected fewer planets than the other method?
   This method requires precise spectroscopy, which is expensive

3. [3 pts] The following question concerns TW Hya.

(a) [1 pt] Identify which image on the image sheet corresponds to this DSO. **Image P**

(b) [1 pt] Which telescope(s) collected data for this spectrum? **ALMA**

(c) [1 pt] Is the protoplanetary disk around TW Hya oriented face-on or edge-on to Earth? **face-on**

4. [2 pts] The following question concerns Image K.
   (a) [1 pt] Which DSO is depicted? **Beta Pictoris**
   (b) [1 pt] In what portion of the EM spectrum was the data collected to make Image K? **IR**

5. [2 pts] The following question concerns Image M.
   (a) [1 pt] Which DSO is depicted? **2M 1207**
   (b) [1 pt] What type of star (e.g., Sun-like star, red giant, white dwarf, etc.) is the central object shown in this image? **Brown dwarf**

6. [5 pts] The following question concerns the Carina Nebula.
   (a) [1 pt] Identify which image on the image sheet corresponds to this DSO. **Image O**
   (b) [1 pt] Which telescope(s) collected data for this image? **Hubble**
   (c) [3 pts] The red, blue, and green in this image correspond to the glow of three different elements. Which elements correspond to which colors? **red: sulfur, green: hydrogen and nitrogen, blue: oxygen**

7. [2 pts] The following question concerns Image Q.
   (a) [1 pt] Which DSO is depicted? **HR 8799**
   (b) [1 pt] Through what exoplanet detection technique (e.g., radial velocity, transit, direct imaging, etc.) were the exoplanets in this system found? **Direct imaging**

8. [4 pts] The following question concerns Image S.
   (a) [1 pt] Which DSO is depicted? **NGC 1333**
   (b) [2 pts] Which telescope collected data for this image? Which range of the EM spectrum did it operate in for this image? **Spitzer, IR (infrared)**
   (c) [1 pt] What do the green streaks and splotches represent? **The glow of cosmic jets blasting away from emerging young stellar objects as the jets collide with the cold cloud material**

9. [9 pts] The following question concerns Image C, which shows $F_{\text{planet}}/F_{\text{star}}$ for one of the DSOs on this year's rules.
   (a) [1 pt] Which DSO does this flux ratio spectrum correspond to? **WASP- 18b**
   (b) [2 pts] Which telescope collected data for this spectrum? Which range of the EM spectrum did it operate in for this image? **Spitzer, IR (infrared)**
   (c) [1 pt] Identify the image (on the image sheet) corresponding to the thermal emission spectrum of this DSO. **B**
   (d) [1 pt] If the DSO was a blackbody with emission spectrum as in the previous part, calculate its temperature. **2850-3050 K**
   (e) [2 pts] This DSO was notable (among other things) for the identification of a certain molecule in its spectrum in 2017. What molecule was it? How did they find out this molecule existed in the atmosphere? **CO, absorption features at 1.6 and 4.5** μm

(f) [2 pts] Explain the general shape of the spectrum in Image C. *Hint: think about blackbody physics.* **The curve is a ratio of two different blackbody spectra - the star at 6400 K and the planet at 3000 K. It's monotonically increasing because blackbodies at a higher temperature emit more at all wavelengths, with a larger discrepancy at shorter wavelengths, near the peak of the higher temperature blackbody. The lower temperature blackbody emits more at longer wavelengths, so the ratio is larger at these wavelengths.**

10. [9 pts] The following question concerns Image J.
    (a) [1 pt] Which DSO is depicted? **AB Aurigae**
    (b) [2 pts] Which telescope(s) collected data for this spectrum? Which range(s) of the EM spectrum did it (they) operate in for this image? **Charis, near-IR; ALMA, sub-mm**
    (c) [2 pts] What do the red disk and blue spirals suggest about this system? **Planet formation**
    (d) [1 pt] What type of object is indicated by the green arrow in this image? **AB Aur b, protoplanet**
    (e) [2 pts] Assuming the object in the previous part orbits around the central star in this DSO with semimajor axis 93 AU, estimate its orbital period in (Earth) years. **579 years**
    (f) [1 pt] Which other image on the image sheet characterizes the central star of this system? **Image D**

11. [11 pts] The following question concerns Luhman 16.
    (a) [2 pts] Identify which image on the Image Sheet corresponds to this DSO and the wavelength range of this image. **Image I, IR**
    (b) [2 pts] Is this DSO moving towards or away from the Solar System? What parameter of the system would you measure to determine this? **Away, radial velocity**
    (c) [2 pts] Which element's spectral lines did researchers use to estimate the age of this system? **Lithium**
    (d) [1 pt] In which part (thin disk, thick disk, core, etc.) of the Milky Way does this system reside? **Thin disk**
    (e) [2 pts] Why did researchers originally hypothesize a potential exoplanet existed in this system? **Perturbations in orbital motion**
    (f) [2 pts] Identify the image depicting a periodogram of one of the components in this DSO and identify a reason for the multiple peaks in this graph. **Image F, differential rotation**

12. [14 pts] The following question concerns Image R.
    (a) [1 pt] Examine the evolutionary track of the $3\,M_\odot$ star. What kind of evolutionary track is this? **Henyey track**
    (b) [1 pt] What kind of pre-main sequence star is this? **Herbig Ae/Be star**
    (c) [1 pt] Examine the evolutionary track of the $0.2\,M_\odot$ star. What kind of evolutionary track is this? **Hayashi track**
    (d) [1 pt] What kind of pre-main sequence star is this? **T Tauri star**
    (e) [2 pts] Now, examine the evolutionary track of the sun-like pre-main sequence star star. What causes the "bend" in the evolutionary track?
    The development of a radiative core/zone

(f) [2 pts] What force(s) are acting to stop the collapse of this star when it reaches the main sequence?
Thermal pressure, radiation pressure (1 pt each)

(g) [3 pts] What force(s) are acting to stop the collapse of this star at a later evolutionary stage (i.e. red giant)?
Thermal pressure, radiation pressure, electron degeneracy pressure (1 pt each)

(h) [2 pts] Now, imagine a large protostar, say of mass $9\,M_\odot$. Why do you think this star is not depicted on this H-R diagram?
Massive stars skip the pre-main sequence phase (or it is too short to be observable), i.e. by the time they are visible, they are already fusing hydrogen

(i) [1 pt] Finally, imagine a very small protostar, say of mass $0.05\,M_\odot$. What kind of object will this protostar become? **Brown dwarf**

Section B (50 points)

1. (a) [2 pts] The combined kinetic energy of the two protons is

$$K = 0\text{ J} + \frac{1}{2} \times 1.67 \times 10^{-27}\text{ kg} \times (1\text{ m/s})^2 = 8.35 \times 10^{-27}\text{ J} \tag{1}$$

The potential energy of the system is

$$U = -\frac{6.67 \times 10^{-11}\text{N m}^2\text{ kg}^{-2} \times \left(1.67 \times 10^{-27}\text{ kg}\right)^2}{1\text{ m}} = -1.86 \times 10^{-64}\text{J} \tag{2}$$

The quantity $K + U > 0$, so the two protons are **gravitationally unbound**.

(b) [2 pts] In order to do this, all we have to do is write N in terms of other variables. The mass of the cloud will be

$$M = V\rho = \frac{4}{3}\pi R^3 \rho \tag{3}$$

If each gas molecule has a mass μ, then the total number of molecules in the gas is

$$N = \frac{M}{\mu} = \frac{4\pi R^3 \rho}{3\mu} \tag{4}$$

Plugging this into our equation for K, we get

$$K = \frac{3}{2} \times \frac{4\pi R^3 \rho}{3\mu} \times k_B T = \frac{2\pi R^3 \rho k_B T}{\mu} \tag{5}$$

(c) [2 pts] Here, all we need to do is replace M with an expression containing R and ρ. Using the result from Equation 3,

$$U = -\frac{3}{5}\frac{G\left(\frac{4}{3}\pi R^3 \rho\right)^2}{R} = -\frac{16}{15}\pi^2 G R^5 \rho^2 \tag{6}$$

(d) [4 pts] Per the virial theorem, which says that $2\langle K\rangle = -\langle U\rangle$ in our system,

$$2\left(\frac{2\pi R^3 \rho k_B T}{\mu}\right) = \frac{16}{15}\pi^2 G R^5 \rho^2 \tag{7}$$

Solving for R gives

$$R = \sqrt{\frac{15 k_B T}{4\pi G \rho \mu}} \tag{8}$$

The cloud will collapse if R is **greater** than this value. We can gain some intuition for this from examining Equation 7. Notice how the left side of the equation, which represents the kinetic energy in the system, scales with R^3, while the right side, which represents the potential energy, scales with R^5. It follows that if we increase R, the right side will increase more quickly than the left side, leading to the gravitational potential energy overpowering the pressure of the cloud.

(e) [2 pts] Clouds are generally denser at the **center** than at the edges. This would make the gravitational potential energy a larger negative number, which in turn makes the Jeans length smaller.

(f) [2 pts] These clouds are so cold that H_2's rotational states are not accessible. The H_2 essentially behaves like a monoatomic gas. This would not make a significant difference observationally.

2. (a) [2 pts] When a cloud of gas and dust collapses under gravity, it often possesses some initial rotation, which gives it angular momentum. The motion in the directions orthogonal to the initial rotation tends to cancel out due to collisions and interactions among particles. This cancellation leads to the formation of a flatter spinning disk, where most of the mass ends up concentrated in this rotating plane. However, since angular momentum is conserved, the disk still spins.

   (b) [2 pts] The material in the disk orbits around the center according to Kepler's Laws. This means that material that is farther away has a lower angular velocity. This differential rotation means that within a region, the part closer to the center will orbit faster than the part farther from the center. This causes the region to "smear", resulting in the shear force.

   (c) [1 pt] The force with the **shortest** timescale will dominate.

   (d) [3 pts] Due to random fluctuations in the disk, sometimes local overdensities will occur. If gravity acts faster than all the other forces, then this overdensity can (might) collapse. However, if the pressure or shear timescales are shorter than that of gravity, the pressure waves and shear forces traveling through the disk will smooth out the overdensity before it is able to collapse.

   (e) [4 pts] In order for the chunk of the disk to collapse, gravity must act on the shortest time scale. In other words, $t_{\text{grav}} < t_{\text{shear}}$ and $t_{\text{grav}} < t_{\text{pr}}$. If we solve for Δr, the first condition becomes

$$\Delta r < \frac{G\Sigma}{\Omega^2} \tag{9}$$

Similarly, the second condition becomes

$$\Delta r > \frac{c_{\text{s}}^2}{G\Sigma} \tag{10}$$

Putting these two inequalities together, we get

$$\frac{c_{\text{s}}^2}{G\Sigma} < \Delta r < \frac{G\Sigma}{\Omega^2} \to \frac{c_{\text{s}}^2}{G\Sigma} < \frac{G\Sigma}{\Omega^2} \tag{11}$$

The inequality can be rewritten, after taking the square root, as

$$Q \sim \frac{c_{\text{s}}\Omega}{G\Sigma} < 1 \tag{12}$$

   (f) [4 pts] As the chunk collapses, it releases gravitational potential energy, so it *also* has to **cool quickly enough**. Otherwise, it will heat up, which will slow (or stop) the collapse.

   (g) [2 pts] Let L represent length, M represent mass, and T represent time. So, c_{s} has dimensions of LT^{-1}, G has dimensions of $\text{L}^3\text{M}^{-1}\text{T}^{-2}$, and Σ has dimensions of ML^{-2}. We need $\lambda \sim c_{\text{s}}^{\alpha} G^{\beta} \Sigma^{\gamma}$ to give us something with dimensions of L. This calls for a system of linear equations:

$$\begin{cases} \alpha + 3\beta - 2\gamma = 1 \\ \beta + \gamma = 0 \\ \alpha - 2\beta = 0 \end{cases} \tag{13}$$

Solving this, we find that $\alpha = 2$, $\beta = -1$, and $\gamma = -1$, so $\lambda \sim c_{\text{s}}^2/G\Sigma$.

   (h) [2 pts] The mass available will be the product of the surface mass density, Σ, and the area of a circle with radius λ:

$$M \sim \pi\lambda^2\Sigma \sim \frac{c_{\text{s}}^4}{G^2\Sigma} \tag{14}$$

3. (a) Planetary blackbody temperature is given by

$$T_p = T_s\sqrt{\frac{R_s}{2d}}, \tag{15}$$

where T_s, R_s are the tempetaure and radius of the parent star and d the distance of the planet from the star. Plugging in values, the "goldilocks" zone is **0.558-1.04** AU. Anywhere in the range **0.5-1.1** AU is acceptable. 1 pt for lower bound, 1 pt for upper bound.

(b) $(1-\alpha)^{1/2}$ (2 pts, all or nothing).

(c) Now

$$T_p = T_s\left(1-\alpha_{\text{eff}}\right)^{1/4}\sqrt{\frac{R_s}{2d}}, \tag{16}$$

where

$$\begin{aligned}\alpha_{\text{eff}} &= \alpha_a + (1-\alpha_a)^2\alpha_s + (1-\alpha_a)^2\alpha_s^2\alpha_a + ...\\ &= \alpha_a + \frac{(1-\alpha_a)^2\alpha_s}{1-\alpha_a\alpha_s}.\end{aligned}$$

3 pts, all or nothing.

(d) $\alpha_{\text{eff}} = 0.904$ (1 pt, all or nothing).

(e) Decrease (1 pt), α_a increases, increasing α_{eff} (1 pt).

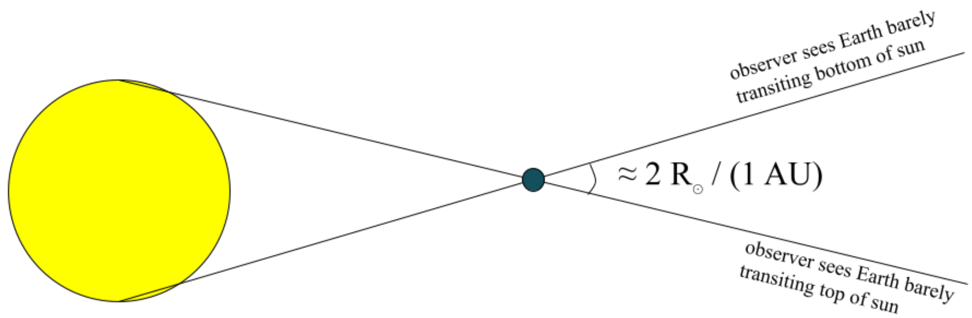
(f) Assume that the blackbody temperature can reach $T_f = 100°$ C and let the fraction of ice at this point be x. Then, we want to solve for x such that

$$\left(\frac{1-\alpha_{\text{eff}}(\alpha_a, x\alpha_{\text{ice}} + (1-x)\alpha_{\text{water}})}{\alpha_{\text{eff}}(\alpha_a, \alpha_s)}\right)^{1/4} = \frac{T_f}{T_i} \tag{17}$$

where $\alpha_{\text{eff}}(\alpha_a, \alpha_s) = 0.904$ (previous part), $T_f = 373$ K, $T_i = 272$ K. Solving, $x \approx 0.62$ so that around 38% of the ice can melt. 2 pts for arriving at Eq. 3 and 2 more points for solving for the correct x ($\pm 5\%$ uncertainty is acceptable).

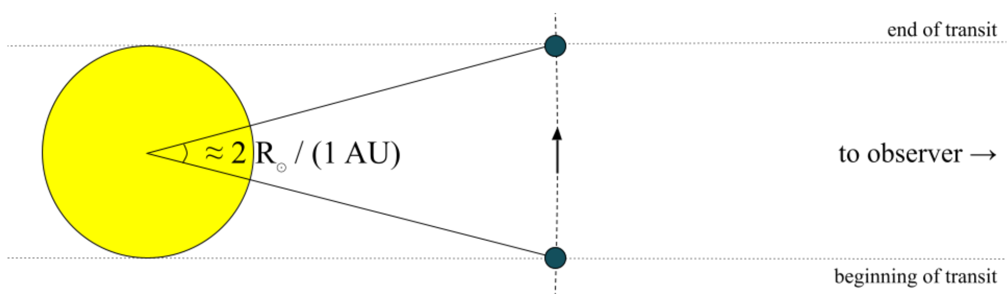
4. (a) The aliens must be located in a band centered on the ecliptic plane that is $\frac{2R_\odot}{1\,\text{AU}} \approx 0.0093$ radians wide (see picture below), making use of the small angle approximation. (2 points)

   Because this angle is small, we can use a planar approximation and calculate that the solid angle that this band takes is $0.0093 \cdot 2\pi$ steradians. The aliens could be located anywhere in the sky, and there are 4π steradians over a sphere, and hence the probability of the Earth and sun having a transiting configuration to a random alien is 0.0047, or 0.47%. (2 pts)



   Note that placing the alien randomly in the sky is mathematically equivalent to having Earth's orbital plane be randomly aligned with respect to a fixed alien observer.

   (b) Earth orbits 2π radians in 365.26 days, and hence it takes $(365.26\,\text{days}) \cdot \frac{2R_\odot/(1\,\text{AU})}{2\pi} = 0.54\,\text{days}$ for Earth to transit the sun. (3 pts)



   (c) From the previous question, we know that the transit duration $T_{trans} \propto T/a$, where T and a are the orbital period and radius, respectively. (1 pt) From Kepler's 3rd law, $T = 1\,\text{yr} \cdot \left(\frac{a}{1\,\text{AU}}\right)^{3/2}$ in the solar system. (1 pt) Therefore, $T_{trans} \propto \frac{a^{3/2}}{a} = a^{1/2}$, and hence $\gamma = 1/2$. (1 pt)

   The constant of proportionality T_0 must be just the transit duration of Earth, which we can see if we plug in $a = 1\,\text{AU}$, so $T_0 = 0.54\,\text{days}$. (1 pt)

   (d) In one year, Earth will spend 0.54 days in a visible transit. Let t_0 be the time of the beginning of Earth's transit. Modulo a year, the time of the beginning of any 30-day continuous observing run can range from $t_0 - 30\,\text{days}$ to $t_0 + 0.54\,\text{days}$, a range of 30.54 days. Since the aliens observe Earth at a random time of the year, the probability of the aliens observing any part of the transit is $30.54/365.26 = 0.084$, or 8.4%. (2 pts)

   (e) Jupiter's transit time is $T_J \cdot \frac{2R_\odot/(a_J)}{2\pi} = 0.00029T_J$. The probability that a transit is observed is $\frac{0.0029T_J + 30\,\text{days}}{T_J} = 0.0029 + \frac{30}{11.86 \cdot 365.25} = 0.0029 + 0.0069 = 0.0098.$, or .98%. (2 pts)

   (f) Assuming that the radiation from Earth is negligible, the transit depth is given by the fractional area of the sun that Earth blocks out, i.e. $\frac{\pi R_\oplus^2}{\pi R_\odot^2} = 0.000084$. (2 pts)

(g) The transit depth is proportional to the square of the planet radius, so Jupiter's transit depth is $(R_J/R_\oplus)^2 = 125$ times deeper than Earth's transit depth. (1 pt)

(h) The planet radius can be inferred from the time it takes for the transit to reach its lowest depth, i.e. the time from the planet's first contact to the second contact, or alternatively third contact to fourth contact. (2 pts) Possible advantages include better measurement of time than transit depth (due to low SNR of transit depth), and not having to assume that radiation from the planet is negligible, as some planets are very hot and may have non-negligible thermal radiation. (2 pt)

(i) Since the transit probability doesn't depend on the planet radius, it is the same as it is for Earth: 8.4%. (1 pt)

(j) The transit probability is the same as for Earth, but the transit depth is the same as Jupiter's, and hence the transit is 125 times more likely to be detected than Earth's transit. (1 pt)

(k) The transit method has a selection bias towards high masses because it is biased towards planets with large radii, because they have deeper transits that are more easily detected within some SNR threshold. (1 pt) The transit method also has a selection bias towards lower periods because the probability of catching a planet in transit roughly scales inversely with its orbital period, as we demonstrated in parts (d) and (e). (1 pt)

(l) The bigger the planet, the bigger the induced Doppler wobble, since $m_1 v_1 = m_2 v_2$. So, there is also a selection bias towards higher-mass planets. (2 pts)

(m) The radial velocity method likely also has a selection bias in period for the same reason: the Doppler wobble caused from planets that are farther away is both weaker in magnitude and longer period, making it harder to detect. (2 pts) However, this bias must be weaker than the bias from the transit method (1 pt), as illustrated by the fact that the planets detected from the radial velocity method have higher periods than the planets detected from the transit method. (1 pt)

Section C: JS9

1. (a) [1 pt] 23
   (b) [1 pt] It is a grating observation.
   (c) [1 pt] There are prominent peaks at 1.8 and 2.0 keV, with a broad maximum at about 3.5 keV
   (d) [1 pt] Wien's Law.
   (e) [1 pt] Assuming maximum emission at 3.5 keV, the temperature is 8.2×10^6 K.
   (f) [1 pt] Erratic small variability for the first half of the observation, followed by a dramatic rise to much higher count rates.
   (g) [1 pt] Steady emission for about half a day, followed by an enormous but brief "pulse" before resuming its previous intensity. Then, after another 20,000 seconds, it starts ringing like a bell!
   (h) [1 pt] Bizarre pulsations for about 20,000 seconds, followed by steady emission for the rest of the observation.
   (i) [1 pt] Broad pulsations, possibly periodic this time. Eyeballing it, a period of ~ 1000 seconds.
   (j) [1 pt] Spike at f = 0.00081 Hz. 1/f = 1234 sec.