Welcome to the UT Invitational 2020! This test is for **Density Lab** in Division B. You will have **50 minutes** to complete this exam.

This exam consists of three sections: **true/false** (Questions 1-25, 50 points), **multiple choice** (Questions 26-40, 45 points), and **long-form questions** (Questions 40-90, 200 points) for a total of **90 questions** and **295 points**. The long-form section consists of a mix of true/false, multiple choice, and short answer questions that are related to a topic/scenario and often build off each other. A couple of the multiple-choice questions have multiple answers, so be sure to select all that apply. Furthermore, some questions ask you to enter your answers in a very specific format. Pay close attention to this, or your answers will not be graded correctly.

Generally, the true/false section is the easiest, followed by multiple choice, and then the long-form questions. However, there are numerous exceptions, so we strongly recommend looking at every question and taking the test in a way that plays to your strengths. Furthermore, we recommend answering every question you can, even if you're not sure about your answer - you might be right, and there's no penalty for guessing!

Density Lab is a unique event in that it incorporates aspects of physics and chemistry that aren't often covered in a normal class in school. We've done our best to show what a beautiful and diverse subject it is through the questions on this exam, ranging from carbon capture to materials science. With that being said, we hope y'all have a wonderful time taking this test!

Note: Questions 82-90 are meant to be especially challenging for a bit of added fun. If you haven't finished other parts of the test, don't worry too much about these!

True/False (2 points each, 50 points total)

1. (2.00 pts) A meter is a unit of length.

- (●) True
- ( ) False

2. (2.00 pts) All substances are denser in the solid state than liquid state.

- ( ) True
- (●) False

3. (2.00 pts) When people refer to "absolute zero", they are referring to a temperature of 0 degrees Celsius.

- ( ) True
- (●) False

4. (2.00 pts) When doing calculations with the Ideal Gas Law, scientists typically use units like Kelvin instead of Celsius.

- (●) True
- ( ) False

5. (2.00 pts) In a solid, molecules can easily move past each other.

- ( ) True
- (●) False

6. (2.00 pts) Typically, gases are denser than liquids.

- ( ) True
- (●) False

7. (2.00 pts) A process in which the temperature remains constant is referred to as isothermal.

- (●) True
- ( ) False

An intensive property is a property of matter that does not change as the amount of matter changes. On the other hand, an extensive property depends on how much matter is there. Use these definitions to help you answer the next 4 questions.

8. (2.00 pts) Mass is an intensive property.

○ True ◉ False

9. (2.00 pts) Density is an intensive property.

◉ True ○ False

10. (2.00 pts) Volume is an extensive property.

◉ True ○ False

11. (2.00 pts) Temperature is an intensive property.

◉ True ○ False

12. (2.00 pts) A molecule containing two atoms, such as $\mathrm{N_2}$, is referred to as “diatomic”.

◉ True ○ False

13. (2.00 pts) Charles' Law states that volume and temperature are directly proportional to each other when all other variables are held constant.

◉ True ○ False

14. (2.00 pts) Gay-Lussac's Law states that pressure and temperature are directly proportional to each other when all other variables are held constant.

◉ True ○ False

15. (2.00 pts) Boyle's Law states that volume and pressure are directly proportional to each other when all other variables are held constant.

○ True ◉ False

16. (2.00 pts) Avogadro's number is about $6.022 \times 10^{23} \frac{1}{\mathrm{mol}}$

◉ True ○ False

17. (2.00 pts) The density of a gas is dependent on temperature.

◉ True ○ False

18. (2.00 pts) In an ideal mixture of gases, the sum of the individual partial pressures equals the total pressure.

◉ True ○ False

19. (2.00 pts) Gases move from low to high pressure.

○ True ◉ False

20. (2.00 pts) A measurement of the area density of an object could have units of $\frac{1}{\mathrm{meters}^2}$

◉ True ○ False

21. (2.00 pts) A scientist wants to measure the number density of atoms in a block of a material. This measurement would be unitless (dimensionless).

○ True ◉ False

22. (2.00 pts) A scientist measures the volume of a material. This measurement could have units of cm^2.

○ True ◉ False

23. (2.00 pts) In an ideal gas, all collisions between molecules are elastic.

◉ True ○ False

24. (2.00 pts)
Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

◉ True ○ False

25. (2.00 pts) In the van der Waals equation of state, the numerical value of a is proportional to the size of the molecules in the gas.

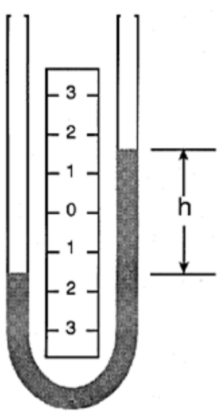
○ True ◉ False

Multiple Choice (3 points each, 45 points total)

26. (3.00 pts) Which of the following gases are monatomic?

○ A) Hydrogen
○ B) Oxygen
◉ C) Xenon
○ D) Nitrogen
○ E) Fluorine

27. (3.00 pts) In the manometer shown below, which side is exposed to a higher pressure?



○ A) Right
◉ B) Left
○ C) Neither

28. (3.00 pts) If a process is said to be "isobaric", it occurs at constant

- [x] A) Pressure
- [ ] B) Volume
- [ ] C) Temperature
- [ ] D) Entropy

29. (3.00 pts) If an object sinks when placed in a liquid, which of the following must be true?

- [ ] A) The object has a high mass
- [ ] B) The object is not as dense as the liquid
- [ ] C) The liquid has a high mass
- [x] D) The object is denser that the liquid

30. (3.00 pts) What are the SI base units of a Pascal?

- [ ] A) (kg*m)/s
- [ ] B) kg/(m*s)
- [ ] C) (kg*m^2)/s^2
- [x] D) kg/(m*s^2)

31. (3.00 pts) The pressure of a gas can depend on which of the following parameters? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) Temperature
- [x] B) Volume
- [x] C) Number of moles

32. (3.00 pts) Which of the following stays constant during a phase change? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) Pressure
- [ ] B) Volume
- [x] C) Temperature
- [x] D) Chemical potential

33. (3.00 pts)
Consider a mixture of two real gases that have have repulsive interactions between them. How would their system pressure compare to the case where both gases were ideal?

- [x] A) The real gas would have a higher pressure
- [ ] B) The real gas would have a lower pressure
- [ ] C) The real gas would have the same pressure as the ideal gas

34. (3.00 pts) If a small amount of salt is added to water, what will happen to the water's boiling point?

- [x] A) It increases
- [ ] B) It decreases
- [ ] C) It stays the same
- [ ] D) None of the above

35. (3.00 pts) Which of the following can cause deviations from ideal behavior in a mixture of two gases?

- ○ A) A decrease in pressure
- ○ B) An increase in temperature
- ◉ C) The presence of intermolecular forces between molecules
- ○ D) Being situated in a large volume

36. (3.00 pts) At a certain temperature and pressure, it becomes impossible to distinguish between a liquid and a gas. What is this point called?

- ○ A) Triple point
- ◉ B) Critical point
- ○ C) Boiling point
- ○ D) Sublimation point

37. (3.00 pts) When a reaction occurs, what value always increases?

- ○ A) The enthalpy of reaction
- ○ B) The Gibbs free energy of the reaction
- ◉ C) The entropy of the universe
- ○ D) The Helmholtz free energy of universe

38. (3.00 pts) Scientists can use the Gibbs free energy of a process to determine whether it is spontaneous, but only under certain conditions. What are these conditions?

- ○ A) Constant temperature and volume
- ◉ B) Constant temperature and pressure
- ○ C) Constant number of moles and volume
- ○ D) Constant entropy

39. (3.00 pts) Which of the following tools could be used to measure the pressure of a system?

(Mark **ALL** correct answers)

- [ ] A) Thermocouple
- [x] B) Manometer
- [x] C) Barometer
- [ ] D) Hygrometer

40. (3.00 pts) Which of these parameters does not affect the pressure of a point at the bottom of some body of water, like a lake or an ocean?

- ○ A) Density
- ○ B) Temperature
- ◉ C) Volume
- ○ D) The height above the water above where you're measuring

Long-form questions (4 points each, 200 points total)

Ideal Gases

Consider a 1 mole of an ideal gas in a 10 L container at 300 K.

41. (4.00 pts) What is 300 K in Celsius?

- ○ A) 300 degrees C
- ○ B) 100 degrees C
- ○ C) 30 degrees C
- ◉ D) 27 degrees C
- ○ E) 0 degrees C

42. (4.00 pts) What is the pressure of the gas inside the container, in atmospheres? *Hint: use 0.08206 (L atm)/(mol K) as the value of the gas constant, R.*

- ○ A) 1.5 atm
- ○ B) 2 atm
- ◉ C) 2.5 atm
- ○ D) 3 atm
- ○ E) 3.5 atm

43. (4.00 pts) Suppose that the temperature of the gas is doubled to 600 K. What happens to the pressure inside the container?

- ◉ A) It doubles
- ○ B) It gets cut in half
- ○ C) It stays the same

44. (4.00 pts) Which of the following best supports your answer to the previous question?

- ○ A) Charles' Law
- ○ B) Avogadro's Law
- ◉ C) Gay-Lussac's Law
- ○ D) Boyle's Law
- ○ E) Le Chatelier's principle

45. (4.00 pts) After more observations, you determine that the molar mass of the gas is 100 grams/mol. What is the mass density of the gas, in kilograms per cubic meter?

- ○ A) 1 kilogram per cubic meter
- ◉ B) 10 kilograms per cubic meter
- ○ C) 25 kilograms per cubic meter
- ○ D) 100 kilograms per cubic meter

Reaction with Ideal Gases

Chemical reactions are a part of our everyday lives. From keeping our atmosphere properly oxygenated to making sure our oceans stay habitable, molecules reacting to create new more complex molecules is essential. Reactions can be very complex, but one form of reaction can be seen below:

$\mathrm{A\,(g)} \longrightarrow \mathrm{2\,B\,(g)}$

This essentially means that one mole of reactant “A” can react to become 2 moles of product “B”. The “g” in parentheses means that both the reactant and product are in the gas phase. The reaction is allowed to run for some time and you find that the concentration of A (Ca) is 0.1 mol/L and the concentration of B (Cb) is 0.08 mol/L.

46. (4.00 pts)
Determine the partial pressure of A (P_A) and the partial pressure of B (P_B) in atmospheres using the Ideal Gas Law, $PV = nRT$. Assume the reactor is run at 373K. Use R = 0.08206 (L*atm)/(mol*K). *Hint: concentration is equal to n/V in the Ideal Gas Law.*

- ◉ A) P_A = 3.06 atm, P_B = 2.45 atm
- ○ B) P_A = 2.45 atm, P_B = 3.06 atm
- ○ C) P_A = 0.79 atm, P_B = 1.41 atm
- ○ D) P_A = 4.22 atm, P_B = 2.78 atm

E) P_A = 1.11 atm, P_B = 0.89 atm

47. (4.00 pts)
Assuming that A and B are the only components in the system, calculate the total pressure of the system in atmospheres. Enter your answer as a decimal between 0 and 10 in the form X.XX without any units.

5.51

48. (4.00 pts)
Knowing that the mole fraction of a component is the partial pressure of that component divided by the total pressure, calculate the moles fraction, x_a, for component A. Enter your answer as a decimal between 0 and 1 in the form 0.XXX. Since this is a mole fraction, there are no units.

0.555

49. (4.00 pts) Assuming you start with only component A and that the reactor is isothermal, how does the pressure of the system change as the reaction progresses?

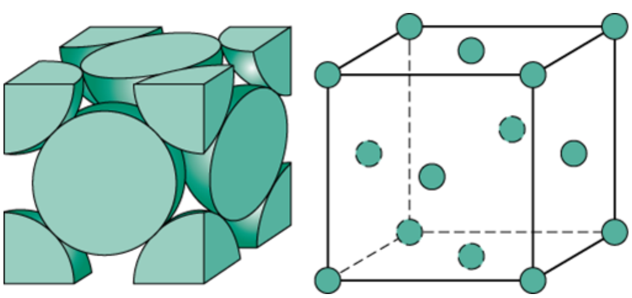
- (selected) A) The pressure increases
- B) The pressure decreases
- C) The pressure stays constant

50. (4.00 pts) Qualitatively explain your answer to the previous question.

Expected Answer: One mole of A makes two moles of B, so the number of moles in the reactor goes up. Since volume and temperature are held constant, that means the pressure must increase.

Unit Cells

In solids, atoms typically arrange themselves into a periodic structure called a lattice. In this question we'll look at silver's lattice structure at room temperature - the face centered cubic unit cell shown below.



51. (4.00 pts)
We can determine the number of atoms in a unit cell by adding the fraction of each sphere contained in the cell. For instance, if an atom is on the corner of the cell, it counts as 1/8 of an atom, and if an atom if centered on one of the faces, then it counts as 1/2 of an atom.

How many atoms are contained in one FCC unit cell?

- A) 2
- (selected) B) 4
- C) 8
- D) 12

The lattice constant a is the length of the side of the unit cell, which is the cube shown in the figure above. The atomic packing factor of a FCC lattice equals the volume of space occupied by atoms within each unit cell divided by the volume of the unit cell. Given that the atomic radius of silver is $144 \times 10^{-10}\mathrm{cm}$,

52. (4.00 pts)
What is the volume, in cm^3, of the silver atoms contained within the unit cell? *Hint: multiply your answer to the previous question by* $\frac{4}{3} * \pi * r^3$.

The volume can be written in scientific notation as such: $A \times 10^B$. In the first blank, give your answer to A as a decimal in the form X.XX. In the second blank, write the exponent, B as a positive or negative integer between -99 and 99.

5.00

-23

53. (4.00 pts)
If $a = 2\sqrt{2} \times R$, what is the volume of the unit cell, in cm^3? *Hint: the volume of a cube with side length a is* a^3.

The volume can be written in scientific notation as such: $A \times 10^B$. In the first blank, give your answer to A as a decimal in the form X.XX. In the second blank, write the exponent, B as a positive or negative integer between -99 and 99.

6.76

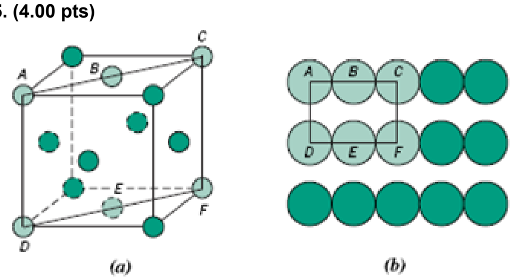
-23

54. (4.00 pts) Based on your answers to the previous two questions, what is the atomic packing factor for the FCC unit cell? *Hint: divide your answer to #52 by your answer to #53.*

Enter your answer as a decimal between 0 and 1 in the form of 0.XX. Since this is a ratio of two volumes, there are no units.

0.74

55. (4.00 pts)



Consider the plane shown in the figure above, where atoms A, C, F, and D are corners. What is its planar density, in units of atoms per square centimeter? Planar density is defined as the number of atoms centered on the plane divided by the area of the plane. *Hint: only count the fraction of the atom that's on the plane, not the entire atom!*

The planar density can be written in scientific notation as such: $A \times 10^B$. In the first blank, give your answer to A as a decimal in the form X.XX. In the second blank, write the exponent, B as a positive or negative integer between -99 and 99.

8.53

14

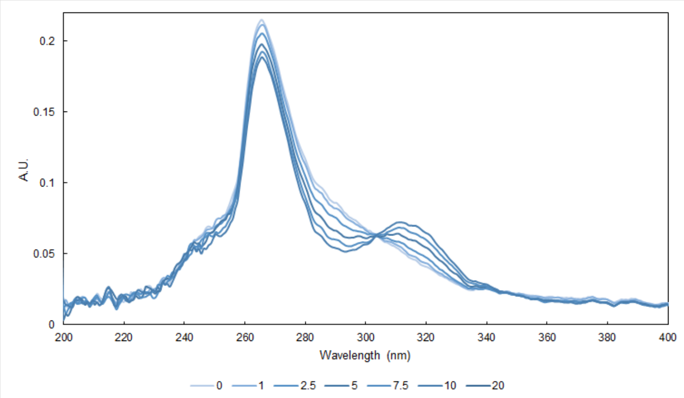
56. (4.00 pts)
Calculate the density of silver in units of g/cm^3 to the nearest tenths place using your answers to #51 and #52, silver's molar mass of 107.8682 g/mol, and Avogadro's Number, 6.022*10^23 atoms/mol.

Enter your answer as a decimal between 10.0 and 99.9 in the form XX.X without any units.

10.6

Spectroscopy

In research labs, UV-VIS spectroscopy is frequently used to determine the concentration of a compound as it reacts with other compounds. This is especially helpful in determining the reaction kinetics (how quickly the compounds react with each other). Shown below is a UV-VIS spectrum obtained in one of UT's labs.



We will focus on the peak at wavelength ~265 nm to analyze our data. The line on the spectrum was set to get darker as time progressed. The y-axis of the spectrum is a unitless value called the absorbance, which will be useful in later calculations. From this, we can see the absorbance decreased with time. To prepare a sample for the UV-VIS, we must first do a set of dilutions to get the sample dilute enough for measurement.

57. (4.00 pts)
Starting with 5 milligrams of compound with a molecular weight of 100 g/mol, calculate the concentration, in mol/L, if you add 10 milliliters of solvent to a vial containing the solid.

Enter your answer as a decimal in the form X.XXX without units.

0.005

58. (4.00 pts)
Another dilution must be done to decrease the concentration of the compound in solution. We want to decrease the concentration to 500 micromolar = 500 * 10^-6 mol/L. What amount of liquid, in mL, should you mix with 1mL of solution made in the previous question to make this concentration? *Hint: conc.(a) * volume(a) = conc.(b) * volume(b). Check units!*

Enter your answer as an integer between 0 and 99 without units in the blank below.

10

We can now use this dilute sample to get absorbance measurements of our compound. The absorbance measurement of the compound in the UV-VIS relates to its concentration via the Beer-Lambert Law, which states that the absorbance (A) is equal to the effective concentration of the compound (c) multiplied by the compound's molar extinction coefficient (ε) and the UV-VIS cell's path length (l). This gives $A = c \times \epsilon \times l$. We do our experiments at a 1 [cm] path length, which is set by the cuvette that we use (shown below). Some pictures of compounds that we put into solution and dilute to make measurements are also shown.

The molar extinction coefficient of one of the compounds is 16,000 [L/(mol*cm)]. Use this for the upcoming calculation.

59. (4.00 pts)
Using the Beer-Lambert Law, calculate the initial effective concentration of the compound [mol/L] in solution. Read the spectrum provided earlier in the problem and use the absorbance [unitless] at the highest peak of the lightest line (time = 0).

Expected Answer: C = 1.31E-5 to 1.44E-5 mol/L

60. (4.00 pts)
This is the initial concentration of our compound in the cuvette, but we also want to calculate the effective concentration of our compound once it has had time to react (the darkest line on the spectrum).

Using the Beer-Lambert Law, calculate the effective concentration of the compound [mol/L] in solution once it has had time to react. Read the spectrum provided earlier and use the absorbance [unitless] at the highest peak of the darkest line (time = 20 minutes).

Expected Answer: 1.06E-5 to 1.19E-5 mol/L

In practice, we use all of the absorbance readings that changed with time and fit the data to an equation. This equation can be a simple linear equation, but it can also have as many as 30 terms. By fitting to an equation, we can get the value of the reaction rate constant k, which will generally tell us how quickly compounds will react with each other (this value is important to characterize a compound's reactivity).

Buoyancy

One day you go to the lake and find a wooden cube floating in water. The wooden cube has a density of 660 kilograms per cubic meter and each side length is 10 centimeters.

61. (4.00 pts) What would be the height of the cube submerged under the water, in cm?

Enter your answer as a decimal between 0.0 and 9.9 in the form X.X without units.

6.6

62. (4.00 pts) Suppose you take the cube out of the lake and place it in oil with a density of 750 kg/m3. What is the buoyant force, in Newtons, of the oil on the cube?

Enter your answer as a decimal between 0.0 and 9.9 in the form X.X without units.

6.5

63. (4.00 pts) As long as the density of the fluid is greater than the density of the wooden cube, the buoyant force of the fluid on the cube will be equal to the weight of the cube.

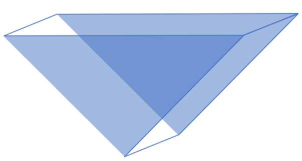
- [x] True
- [ ] False

64. (4.00 pts)
What percent of the cube's volume is under the surface of the oil?

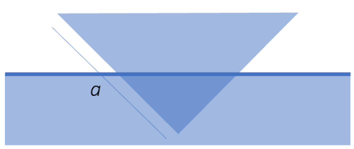
Enter your answer as an integer between 0 and 100 followed immediately by a percent sign (%). For example, 20% and 10% would be valid answers, but 0.20 and 10 would not.

68%

65. (4.00 pts)



the wedge outside of fluid



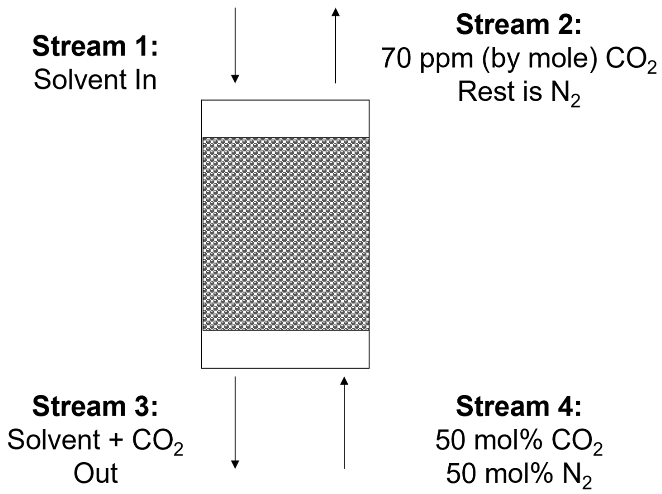
front view in the fluid

Consider a wedge with density ρ_w floating in a fluid of density ρ_f. The base of the wedge is a right, isosceles triangle with side length a, as shown above. What is the depth, d, of the wedge in the fluid?

- [ ] A) $d = a \times \sqrt{\rho_w/\rho_f}$

- [x] B) $d = a \times \sqrt{\rho_w / 2\rho_f}$
- [ ] C) $d = a \times \sqrt{\rho_f / \rho_w}$
- [ ] D) $d = a \times \sqrt{\rho_f / 2\rho_w}$
- [ ] E) $d = a \times (\rho_f / 2\rho_w)$

Carbon Capture



You're a freshly graduated chemical engineer researching carbon capture, the process of preventing carbon dioxide emissions from escaping into the atmosphere. As part of your research, you've designed a special "absorber", as shown in the schematic above. The carbon dioxide entering the absorber is transferred to a special solvent, resulting in the gas leaving the absorber having a very low carbon dioxide content.

66. (4.00 pts)
Since companies often measure their carbon dioxide emissions by mass, a fellow engineer asks you to give the composition of Stream 4 by mass, not by mole. What is the mass fraction of carbon dioxide in Stream 4? *Hint: the molar masses of carbon dioxide and nitrogen gas are 44 g/mol and 28 g/mol, respectively.*

Enter your answer as a decimal between 0 and 1 in the form 0.XXXXX without any units.

0.61111

67. (4.00 pts) Likewise, what is the mass fraction of carbon dioxide in Stream 2?

Enter your answer as a decimal between 0 and 1 in the form 0.XXXXX without any units.

0.00011

68. (4.00 pts)
While testing your absorber, you set the molar flowrate of Stream 4 to 1000 mol/s. What is its mass flowrate, in kg/s? Round your answer to the nearest whole number.

Enter your answer as an integer in the form XX between 10 and 99 without units.

36

69. (4.00 pts)
What percent, by mass, of the carbon dioxide in Stream 4 is removed?

Enter your answer as a decimal in the form XX.XXX followed immediately by a percent sign (%). For example, 20.012% and 10.921% would be valid answers, but 0.20012 and 10.9% would not.

99.993%

70. (4.00 pts)
One of the most important parts of your research is choosing the right solvent. Currently, you've designed this system such that the carbon dioxide is physically dissolved into the solvent. Compare the strength of the intermolecular forces between the carbon dioxide and the solvent in Stream 3 and the carbon dioxide and nitrogen in Stream 4.

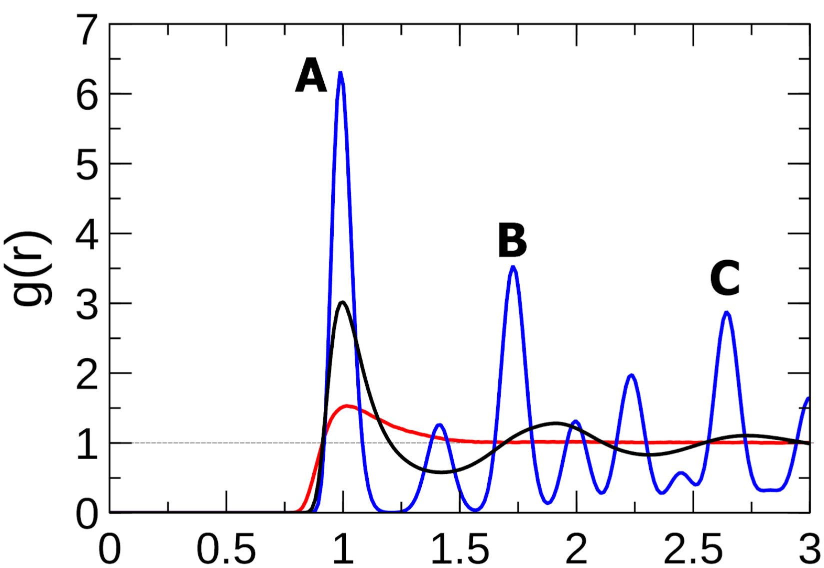
Expected Answer: The IMFs between carbon dioxide and solvent molecules are stronger than the IMFs between carbon dioxide and nitrogen.

71. (4.00 pts)
Your supervisor notes that if the carbon dioxide had a chemical reaction with the solvent, you may be able to capture more of it. If you were designing a chemical absorption unit, would you want the reaction between the carbon dioxide and solvent to be very fast or very slow? If the reaction was very fast, what would be limiting the rate at which you absorb carbon dioxide?

Expected Answer: We would want the reaction to be very fast. If the reaction is very fast, then only the diffusion of the carbon dioxide through the gas layer would limit the absorption.

RDF

A **radial distribution function** (rdf) describes how the density of a system of particles changes depending on where you are. Typically, the x-axis represents the distance from the center of a reference particle, usually in terms of the radius of the particles. The y-axis is a little more complicated, but it is roughly proportional to the number of particles at that distance from the center of the reference particle.



The above plot shows the radial distribution functions for the solid, liquid, and gas forms of the same substance. Assume that each molecule in this substance is a sphere of radius R
.

72. (4.00 pts) In all phases, the value of the radial distribution function between $x = 0$ and $x = 1$ is effectively 0. Why would this be the case?

Expected Answer: Between x=0 and x=1, you're inside the reference particle. Two particles can't be inside each other, so the rdf would be 0.

73. (4.00 pts)
The first three peaks of the blue curve have been labelled with the letters A, B, and C. Which peak represents the number of nearest neighbors the reference molecule has in this phase?

- [x] A) A
- [ ] B) B
- [ ] C) C

74. (4.00 pts) Which curve shows this substance in its solid form?

- [x] A) Blue
- [ ] B) Black
- [ ] C) Red

75. (4.00 pts) Which curve shows this substance in its gaseous form?

- [ ] A) Blue
- [ ] B) Black
- [x] C) Red

76. (4.00 pts) Qualitatively, explain the reasoning you used to answer the previous two questions

Expected Answer: Sharper peaks in the rdf correspond to phases with more structure and predictability. In a solid, the atoms are usually arranged in a repeating crystal structure and can't easily move. However, in a gas, the atoms can move around easily and there is no ordered structure.

Boltzmann Distributions, and some light quantum mechanics

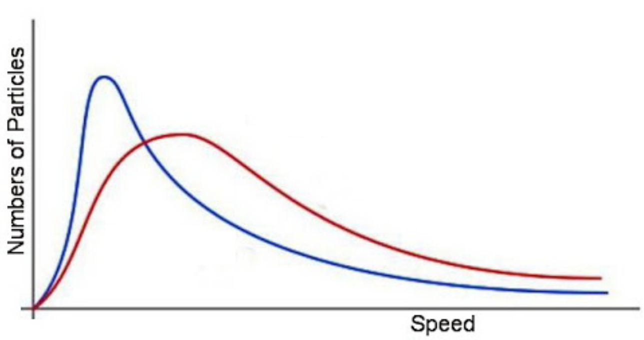
Consider an ideal gas in a container with a movable piston initially at temperature T_1 and pressure P_1. Suddenly, the external pressure the container is exposed to is lowered to a pressure much lower than P_1.

77. (4.00 pts) Will the volume of the gas increase or decrease?

- [x] A) Increase
- [ ] B) Decrease

78. (4.00 pts) Is the expansion/contraction of the gas reversible or irreversible?

- [ ] A) Reversible
- [x] B) Irreversible



After the gas has equilibrated again, you find that it is at a lower temperature than before (we'll call this new temperature T_2). The figure above shows Boltzmann distributions for the gas at T_1 and T_2.

79. (4.00 pts)
The ____ Boltzmann distribution corresponds to the gas at high temperature (T_1), while the ____ Boltzmann distribution corresponds to the gas at low temperature (T_2).

- (selected) A) Red, blue
- B) Blue, red

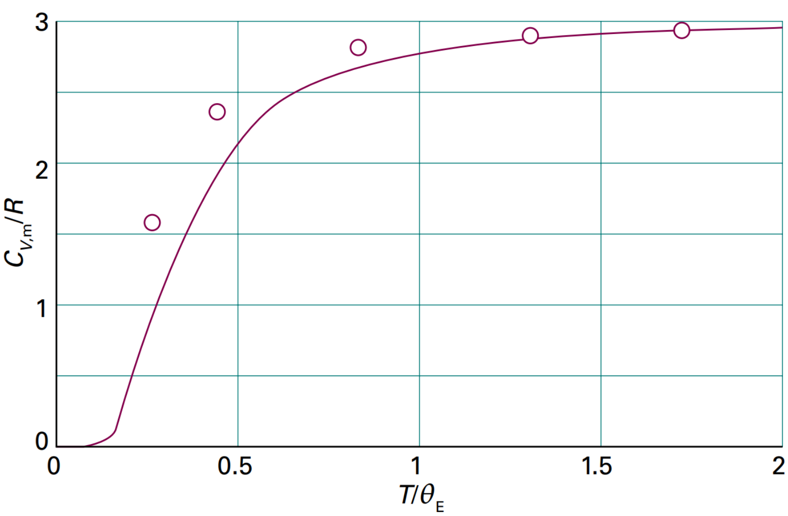
80. (4.00 pts) The area under the red curve is greater than the area under the blue curve.

- True
- (selected) False

81. (4.00 pts) The total energy of an ideal gas (i.e. the sum of its potential and kinetic energies) depends only on its temperature. In your own words, explain why this is the case.

Expected Answer: For this, we can think about what goes into energy: intramolecular forces (vibrations and rotations, especially), and intermolecular forces (Coulombic potentials, dispersion forces, etc.). An ideal gas experiences no intermolecular forces; it should essentially be a point particle zipping around and occasionally hitting its neighbors. Taking this picture further, you would also expect it to lack intramolecular forces (i.e. there are no bonds to vibrate or rotate). All that's left is kinetic energy, which we know is tied to temperature; therefore, we'd expect our total energy (i.e. our kinetic energy) to depend only on temperature, rather than temperature and, say, pressure.

82. (4.00 pts)
The gas is now moved to a special experimental setup, where it is cooled to very low temperatures. Classical mechanics stipulates that the constant-volume heat capacity of a monatomic ideal gas, C_v, is $3R$ at all temperatures. However, when this gas is cooled to very low temperatures, experiments show that its heat capacity decreases sharply, as shown by the figure below:



Einstein improved our understanding of this problem by proposing that each gas molecule was an oscillator that was limited to having certain energies. Qualitatively, explain how this theory could explain the general shape of experimental data shown above. *Hint: think about Boltzmann distributions - at low temperatures, are our oscillators likely to get excited?*

Expected Answer: Based on the Boltzmann distribution, each oscillator is unlikely to be excited to high energies, and at low temperatures, few oscillators can be excited at all. As a consequence, because the oscillators cannot be excited, the heat capacity decreases, and eventually falls to zero.

83. (4.00 pts)
Finally, a single gas molecule (i.e. an atom, since the gas is monatomic) is magically isolated from the initial sample and placed in a small container, where it is cooled to a fraction of a Kelvin above absolute zero. Another prediction of classical mechanics is that at absolute zero, the energy of a system goes to 0. However, experiments show that even at absolute zero, our lone gas molecule has a small amount of energy, called its **zero-point energy**.

The Heisenberg Uncertainty Principle, $\Delta p_x \Delta x \geq \frac{h}{4\pi}$, states that the more we know about a particle's position, the less we know about its momentum (and therefore its energy). Using this concept, qualitatively explain why zero-point energy would exist. *Hint: the isolated gas molecule is in a small box - what does this imply about the range of possible energies it could have?*

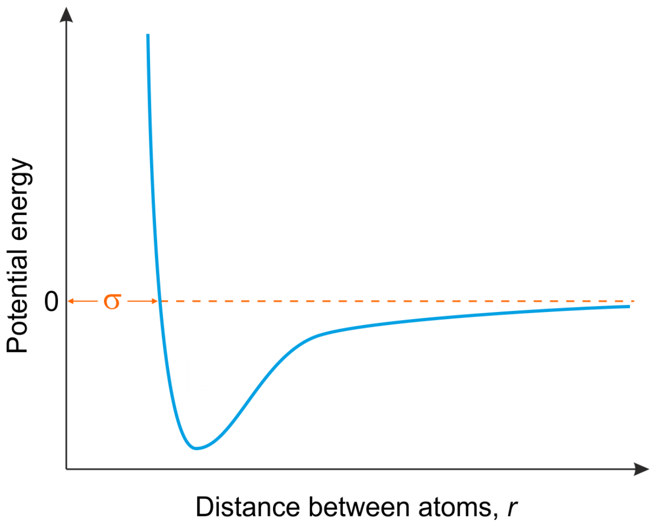
Expected Answer: For a particle confined to a box, we know some stuff about its position (i.e. Δx has a finite value), therefore Δpx cannot be zero, as that would violate the uncertainty principle. Δpx cannot be zero, so the momentum (and therefore energy) CANNOT be a single value (0). Therefore, the kinetic energy of the particle cannot be just zero - it must be some range of values that includes numbers other than 0.

Lennard-Jones Potentials

The potential energy between two molecules in a gas can be described by the famous Lennard-Jones potential:

$U(r) = 4\epsilon\left(\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right)$

where $U(r)$ is the potential energy and ϵ and σ are constants that depend on the gas. The graph of the potential is shown below and intersects the x-axis once at $r = \sigma$.



84. (4.00 pts) For $r < \sigma$, the potential energy is positive. Does this represent attraction or repulsion between the two molecules?

- A) Attraction
- B) Repulsion

85. (4.00 pts) Give a qualitative explanation for your answer to the previous question.

Expected Answer: When two atoms are too close to each other, we'd expect their positive nuclei to interact repulsively, and thus create a positive potential (i.e. it takes energy to push them closer together).

86. (4.00 pts) As the value of ϵ increases in our equation, what happens to the minimum value of the graph?

- A) It becomes a smaller negative number (the "well" becomes shallower)
- B) It becomes a larger negative number (the "well" becomes deeper)

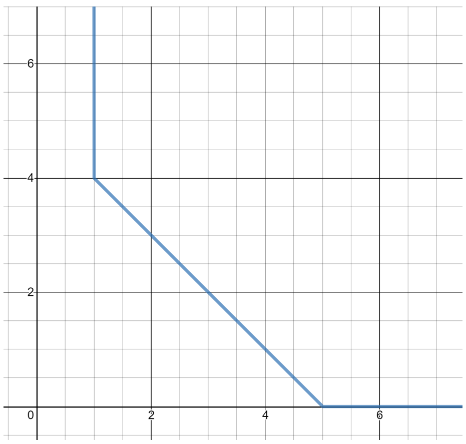
87. (4.00 pts) Would you expect helium or argon to have a larger value of σ?

- ○ A) Helium
- ◉ B) Argon

88. (4.00 pts) Would you expect helium or argon to have a larger value of ϵ?

- ○ A) Helium
- ◉ B) Argon

Doing numerical calculations with the Lennard-Jones potential can be a bit difficult, so sometimes, chemists simplify its shape using piecewise functions. Consider the model for potential energy shown below, where the x and y-axes are both in arbitrary units:



89. (4.00 pts)
Would this model be better at describing the behavior of helium or water? Disregard the numbers on the axes - just focus on the shape of the potential and its implications about the attractive and repulsive forces.

- ◉ A) Helium
- ○ B) Water

90. (4.00 pts)
The **Boyle temperature** is the temperature at which on average, the attractive and repulsive forces between particles in a gas cancel out. Is it possible for a gas modeled using the piecewise potential shown above to have a Boyle temperature? Explain your answer qualitatively.

Expected Answer: This potential will NOT exhibit a Boyle temperature. The Boyle temperature represents the temperature at which the attractive and repulsive forces cancel out on average. However, a gas following this model would have no attractive forces, only repulsive forces. As a result, it’s impossible for it to cancel out, so it’s impossible for this gas to exhibit a Boyle temperature.

We hope you had a wonderful time taking this test! Feel free to email adityashah108@gmail.com if you have any questions about the exam or event.