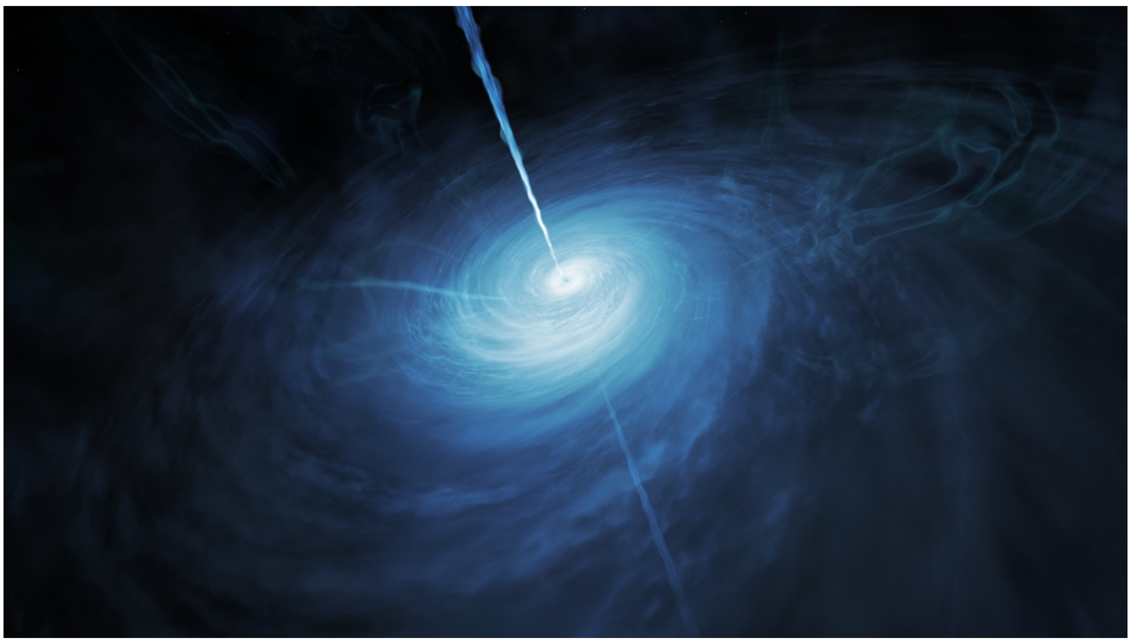
Science Olympiad
Astronomy C
UT Invitational

October 23, 2020



Directions:

- Write **all answers on the lines on the answer pages**. Any marks elsewhere will not be scored.
- Make sure to put your **team name** and **team number** at the top of all answer pages.
- Do not worry about significant figures. Just make sure to **use 3 or more in your answers**.
- Best of luck! And may the odds be ever in your favor.

Written by:
Aditya Shah, *adityashah108@gmail.com*; **Antonio Frigo**, *frigo.antonio.mct@gmail.com*
April Cheng, *aprilcheng0420@yahoo.com*; **Dhruva Karkada**, *dkarkada@gmail.com*
Pranit Mohnot, *pranit_mohnot@berkeley.edu*; **Robert Lee**, *robertyl@ucla.edu*
Sahil Pontula, *sahil31415@gmail.com*

Section A: Multiple Choice

For the following questions, select the correct answer (or correct answers, if specified) for each question. The first several questions involve reading a passage and selecting the right word to complete the blank, which corresponds to a question number. Questions are worth 1 point each, unless otherwise specified, for a total of 69 points.

One of the most remarkable things about astronomy is its scale and its implications on our place in the universe. The average distance between the Sun and the Earth is 1 AU, or 1 __(1)__ It would take a modern jet about 20 years to travel that distance! Even if we travelled at the speed of light, it would still take over 4 years to reach the closest star to our Solar System, __(2)__ .

There are over 100 million stars in our galaxy, __(3)__ , which is a __(4)__ . At the center of our galaxy, astronomers have discovered a __(5)__ . Our galaxy is a part of a group of galaxies called the __(6)__ , which is in turn a part of the Virgo Supercluster. The Virgo Supercluster is estimated to have a diameter of over 100 million light years!

1. A. arbitrary unit
   B. astronomical union
   C. astronomical unit
   D. none of the above
2. A. Sirius A
   B. Proxima Centauri
   C. Betelgeuse
   D. Rigel
3. A. the Milky Way Galaxy
   B. the Andromeda Galaxy
   C. NGC 17
   D. the LMC
4. A. spiral
   B. barred-spiral
   C. elliptical
   D. irregular
5. A. supermassive black hole
   B. alien civilization
   C. globular cluster
   D. neutron star
6. A. Virgo Group
   B. Solar System
   C. Andromeda Group
   D. Local Group

The __(7)__ (abbreviated CMB) is blackbody radiation originating from the very early universe. Its spectrum has an effective temperature of about __(8)__ , which peaks at a wavelength of __(9)__ according to Wien's displacement law. The radiation in fact used to have an effective temperature of about __(10)__ when it was first emitted at z=1100, but the light has been strongly redshifted due to __(11)__ . The blackbody radiation we see today originates from __(12)__ , but ever since __(13)__ , the photons have been freely travelling through the universe. The radiation is pretty much uniform across the sky, except for minor fluctuations which are formally known as __(14)__ . These fluctuations are often studied using __(15)__ , which studies the magnitude of fluctuations as a function of __(16)__ .

7. A. central microwave blackbody
   B. cosmic microwave background
   C. cosmological massive baryon
   D. cosmological massive blackbody

8. A. 3 K
   B. 300 K
   C. 3 °C
   D. 300 °C

9. A. 1.9 nm
   B. 1.9 mm
   C. 1.9 cm
   D. 1.9 m

10. A. 3000 K
    B. 300 K
    C. 30 K
    D. 3 K

11. A. time dilation
    B. gravitational lensing
    C. the doppler effect
    D. the expansion of the universe

12. A. primordial black holes
    B. an opaque plasma
    C. the cosmological constant
    D. the big bang

13. A. the first stars formed
    B. dark matter domination
    C. reionization
    D. photon decoupling

14. A. anharmonicities
    B. anisotropies
    C. Heisenberg uncertainties
    D. the inflaton field

15. A. Weierstrass functions
    B. symplectic geometry
    C. multipole expansion
    D. diffeomorphic geometry

16. A. the tangent bundle
    B. the Lagrangian
    C. Lipschitz bounds
    D. angular size

Gravitational waves are a consequence of __(17)__ , which was developed by Einstein in the early 1900s. It predicts that two orbiting bodies emit __(18)__ . The strength of this emission increases with __(19)__ and decreases with __(20)__ . We have built sensors to detect gravitational waves; the two in the United States are located in the cities __(21)__ . These instruments are light interferometers, much like the interferometer that was famously used to show that __(22)__ . Due to the weakness of gravitational waves, the data collected by these instruments have very low __(23)__ . Nonetheless, scientists have been able to detect many "chirps", which represent __(24)__ .

17. A. the theory of special relativity
    B. the theory of general relativity
    C. the theory of modified Newtonian dynamics (MOND)
    D. the cosmological constant

18. A. ripples in spacetime
    B. plane waves in the inflaton field
    C. blackbody radiation in the gravitational field
    D. heavy leptons and pi mesons

19. A. quantum coherence
    B. distance from the source
    C. orbital angular velocity
    D. temperature

20. A. quantum coherence
    B. distance from the source
    C. orbital angular velocity
    D. temperature

21. A. Hanford and Livingston
    B. San Agustin and Caltech
    C. Phoenix and Socorro
    D. Santo Stefano and Pisa

22. A. light is inherently quantum mechanical
    B. there is no ether
    C. Newtonian gravity is wrong
    D. Maxwellian electromagnetism is wrong

23. A. multimodality
    B. kurtosis
    C. bias/variance tradeoff
    D. signal-to-noise ratio

24. A. the formation of supermassive black holes
    B. starquakes on a neutron star
    C. rotating Kerr black holes
    D. mergers between compact objects

Stellar evolution describes how stars change during their lifetimes. All stars are "born" from a collapsing cloud of gas and dust, often called a __(25)__ . This process forms a __(26)__ . Eventually, the cloud reaches an equilibrium, and the center becomes hot enough to allow for the fusion of hydrogen, becoming what is known as a __(27)__ .

A main-sequence star evolves into a __(28)__ when all the hydrogen in its core is consumed. The star's core contracts and heats up, while its outer layers do the opposite. If the star is massive enough, __(29)__ fusion in the core begins, usually surrounded by a shell of hydrogen fusion. If a star is massive enough, it can go on to fuse even heavier elements. Typically, even the most massive stars can only fuse elements up to __(30)__ , because that's when the binding energy per nucleon is maximized.

25. A. globular cluster
    B. molecular cloud
    C. protoplanetary disk
    D. circumstellar disk

26. A. pre-star
    B. star cluster
    C. protostar
    D. open star

27. A. main-sequence star
    B. red dwarf
    C. white dwarf
    D. neutron star

28. A. main-sequence star
    B. red giant
    C. red supergiant
    D. red dwarf

29. A. helium
    B. carbon
    C. oxygen
    D. uranium

30. A. carbon
    B. iron
    C. uranium
    D. plutonium

A high-mass star ends its life with a powerful explosion called a __(31)__ . If the progenitor (original) star is between about 8 and 30 solar masses, it will leave behind a __(32)__ , which is supported by __(33)__ . If the progenitor is even more massive, it can form a __(34)__ .

In contrast, a star of relatively low mass—such as our own Sun—ends its evolution by gently expelling its outer layers into space. These ejected gases form a glowing cloud called a __(35)__ . The burned-out core that remains is called a __(36)__ , which is held up by __(37)__ . White dwarfs can also become a supernova if it accretes enough mass to exceed the __(38)__ .

31. A. quasar
    B. gamma-ray burst
    C. gravitational wave
    D. supernova

32. A. black hole
    B. neutron star
    C. white dwarf
    D. red dwarf

33. A. radiation pressure
    B. electron degeneracy pressure
    C. neutron degeneracy pressure
    D. gravitational repulsion

34. A. black hole
    B. quasar
    C. active galactic nucleus
    D. blue hypergiant

35. A. stellar halo
    B. galactic halo
    C. planetary nebula
    D. circumstellar disk

36. A. white dwarf
    B. neutron star
    C. black hole
    D. red dwarf

37. A. electron degeneracy pressure
    B. neutron degeneracy pressure
    C. gravitational repulsion
    D. quantum entanglement

38. A. TOV Limit
    B. Einstein Limit
    C. Chandrasekhar Limit
    D. Eddington Limit

To help visualize different types of stars and stellar evolution, astronomers often plot stars on a __(39)__ (HR) diagram. Usually, HR diagrams have __(40)__ on the x-axis and __(41)__ on the y-axis. As a result, different classes of stars would appear in different locations. For example, white dwarfs appear in the __(42)__ , while red supergiants would appear in the __(43)__ .

39. A. Heliotropic Radiation
    B. Hendricks-Rammer
    C. Hayashi-Rhodes
    D. Hertzsprung-Russell

40. A. temperature
    B. pressure
    C. luminosity
    D. radius

41. A. temperature
    B. pressure
    C. luminosity
    D. radius

42. A. top left
    B. bottom left
    C. top right
    D. bottom right

43. A. top left
    B. bottom left
    C. top right
    D. bottom right

The remainder of this section will not be "fill-in-the-blank" like the earlier questions.

Use the following information to answer the next five (5) questions.

Two extraordinarily rich friends are comparing their telescopes. One of them has an x-ray telescope in space, while the other has a ground-based radio telescope.

44. Which portion of the electromagnetic spectrum (gamma-ray or radio) has a higher frequency?
    - A. Gamma-ray
    - B. Radio
45. Which portion of the electromagnetic spectrum (gamma-ray or radio) has photons of a higher energy?
    - A. Gamma-ray
    - B. Radio
46. True or false: the radio telescope must be ground-based since the Earth's atmosphere blocks incoming radio waves
    - A. True
    - B. False
47. One of the friends claims that their telescope has a higher angular resolution, so it is better at resolving objects. Are they correct?
    - A. Yes
    - B. No
48. It turns out that both of their telescopes have the exact same angular resolution. Which one is bigger?
    - A. Gamma-ray
    - B. Radio

Use the following information to answer the next four (4) questions.

Consider 3 stars, A, B, and C, each with varying temperatures and radii. Star A has a temperature of T and a radius of R. Star B has a temperature of $2T$ and a radius of R. Star C has a temperature of $T/2$ and a radius of $2R$.

49. Which star has the highest luminosity?
    - A. A
    - B. B
    - C. C
50. How many times more luminous is Star A than Star C?
    - A. 2
    - B. 4
    - C. 8
    - D. 16
    - E. None of the above
51. Each star's spectrum will peak at a certain wavelength, as predicted by Wien's Law. Which star's spectrum will peak at the longest wavelength?
    - A. A
    - B. B
    - C. C
52. Suppose that Star B is twice as far as Star A from Earth. What is the ratio of the intensity of light we receive from Star B to that of Star A?
    - A. 1/4
    - B. 1/2
    - C. 1
    - D. 4
    - E. 16

---

53. Which of the following lists the order of the main sequence spectral types from hottest to coolest?
    A. OBAFGKM
    B. ABFGKMO
    C. BOGAFMK
    D. ABCDEFG

54. How do stars produce energy?
    A. Chemical reactions
    B. Nuclear reactions
    C. Release of gravitational potential energy
    D. Electron degeneracy pressure

55. The most luminous stars are ____ and ____ .
    A. big, hot
    B. big, cool
    C. small, hot
    D. small, cool

56. Lightyears are a measure of ____ .
    A. distance
    B. energy
    C. speed
    D. time

57. Luminosity is a measure of ____ .
    A. energy
    B. temperature
    C. distance
    D. power

58. Stars spend most of their lives on the ____ on an HR diagram.
    A. Main-sequence
    B. Red giant branch
    C. Asymptotic branch
    D. Instability strip

59. A star's temperature
    A. Increases towards the center.
    B. Decreases towards the center.
    C. Is fairly constant throughout the star.
    D. Increases towards the center, until the core, which is relatively mild.

60. The balance between a star's inward pull of gravity and its outwards pressure gradient is called
    A. Stellar equilibrium
    B. Hydrostatic equilibrium
    C. Eddington limit
    D. Stellar buoyancy

61. *The planets in our Solar System orbit the Sun in an ellipse, with the Sun as one of the foci.* This is Kepler's ____ Law.
    A. First
    B. Second
    C. Third
    D. Fourth

62. $P^2 \propto a^3$. This is Kepler's ____ Law.
    A. First
    B. Second
    C. Third
    D. Fourth

63. What is the Sunyaev-Zel'dovich effect?
    A. The distortion of the CMB due to gravitational lensing
    B. The distortion of spacetime due to the CMB
    C. The distortion of the CMB due to Compton scattering
    D. The distortion of spacetime due to gravitational interactions

---

The next three (3) question are worth **two points**.

Each of the following scenarios occurs in a universe in which one physical law, principle, or fact has been changed. Your task is to determine how this difference affects reality. You may assume (highly unrealistically) that all other principles and laws remain unchanged. Select the answer which is most directly linked to the change.

64. The force of gravity is proportional to r^{-3}.
    A. Atoms no longer can exist, as they fly apart due to the lack of internal attraction.
    B. Neutrinos can fly through much more matter on average before getting stopped.
    C. Most planets are rocky and extremely dense.
    D. The speed of light is no longer constant.

65. The ratio of hydrogen to helium in the universe is 1:1. The mass of the universe is remains the same.
    A. Galaxies are substantially larger on average.
    B. Galaxies are substantially redder on average.
    C. Core-collapse supernovae explosions do not occur at all.
    D. The mass of silicon in the universe would remain the same.

66. The binding energy per nucleon peak occurs at neon.
    A. The composition of the ISM does not change.
    B. Classical novae occur more frequently.
    C. The Sun would explode as a supernova.
    D. Supernovae occur more frequently.

Section B: Deep-Sky Objects

Use the Image Sheet to answer the following questions. Questions are worth 2-3 points each, for a total of 66 points.

The next three (3) questions refer to DLA0817g.

1. (2 points) Which image shows DLA0817g?
   A. Image 2
   B. Image 4
   C. Image 7
   D. Image 11
2. (2 points) Which region of the electromagnetic spectrum was this image taken in?
   A. Gamma-ray
   B. X-ray
   C. Infrared
   D. Radio
3. (2 points) What type of object is DLA0817g?
   A. Quasar
   B. Disk Galaxy
   C. Nebula
   D. We don't know yet

The next five (5) questions deal with Image 1.

4. (2 points) Which DSO is shown in Image 1?
   A. NGC 2623
   B. JKCS 041
   C. H1821+643
   D. 3C 273
5. (2 points) Which region of the electromagnetic spectrum was this taken in?
   A. X-ray
   B. Ultraviolet
   C. Optical
   D. Infrared
6. (2 points) What event is occurring?
   A. Galactic formation
   B. Galactic merger
   C. Star system merger
   D. Galactic death
7. (2 points) Does the core of the object shown have a higher or lower rate of star formation than an average galaxy? Why?
   A. Higher, the event has compressed some gas and dust
   B. Higher, the event caused stars to collide, resulting in them collapsing and joining the ISM
   C. Lower, the event caused gas and dust to heat up, making harder for molecular clouds to collapse
   D. Lower, the increase in entropy due to the event has made it energetically unfavorable for star formation to occur
8. (2 points) Why is this object important in understanding the future of our own galaxy?
   A. The Milky Way is very young, so star formation rates are very high here as well
   B. The supermassive black hole at the center of the Milky Way is unusually massive, which could cause the galaxy to become destabilized
   C. Scientists anticipate that the Milky Way Galaxy will fragment due to dark matter at the edges of its spiral arms within the next 10 billion years
   D. A similar galactic merger will occur between the Milky Way Galaxy and the Andromeda Galaxy within the next 4-5 billion years

---

The next five (5) questions refer to GW151226.

9. (2 points) Which image shows GW151226?
   A. Image 3
   B. Image 5
   C. Image 7
   D. Image 12

10. (2 points) What type of event was GW151226?
    A. Merger of two objects
    B. Type II supernova
    C. Type Ia supernova
    D. A binary star system suddenly and violently shedding its outer layers

11. (2 points) Where in the "celestial sphere" did GW151226 occur? I.e. give the right ascension and declination
    A. 05h 55m 10.30536s, $+07°24' 25.43''$
    B. 04h 36m 45.59127s, $-62°04' 37.80''$
    C. 18h 39m 02.3709s, $-06°05' 10.54''$
    D. We don't know for sure.

12. (2 points) What does GW151226 tell us about the prevalence of its class of DSO?
    A. They may be more common than previously thought.
    B. They may be less common than previously thought.
    C. They are likely as common as previously thought.
    D. It provided no significant evidence as to the prevalence of binary black holes.

13. (3 points) Given that the initial total mass of the progenitor system for GW151226 was 21.7 solar masses, and that the total mass of the resulting system was 20.8 solar masses, how much energy (in Joules), was released in the event? (Provide answer to one decimal place)

The next three (3) questions refer to H2356-309.

14. (2 points) What type of galaxy is H2356-309?
    A. Seyfert I
    B. Seyfert II
    C. Starburst
    D. BL Lacartae

15. (2 points) H2356-309 provided evidence of which of the following?
    A. WHIM
    B. Dark Matter
    C. Both of the above
    D. Neither of the above

16. (3 points) Justify your answer to the previous question. If you said that H2356-309 provided evidence for something (even if it was "neither of the two"), explain the physical process through which it could occur.

One of the DSOs on your list is the farthest known galaxy cluster from Earth. The next five (5) questions refer to this DSO.

17. (2 points) What is the name of this DSO?
    A. Abel 383
    B. JKCS 041
    C. GOODS-S 29323
    D. MACS J0717.5+3745

18. (2 points) Which image shows this DSO?
    A. Image 5
    B. Image 6
    C. Image 8
    D. Image 10

19. (2 points) What is the redshift of this DSO?
    A. 1.1
    B. 0.8
    C. 1.8
    D. 2.3

20. (2 points) Observation of this galaxy cluster can help answer which of the following questions?
    A. Will the Milky Way become a quasar again?
    B. How old is the Milky Way Galaxy?
    C. What is the oldest known galaxy?
    D. When did most galaxies stop forming stars at very high rates?

21. (3 points) Galaxy quenching occurs when a galaxy loses cold gas. It is observed that galaxies within this cluster are quiescent at a rate much higher that of "lone" galaxies from the same time period. What does this say about galaxy quenching?

The next three (3) questions deal with the Chandra Isotropic Universe Survey.

22. (2 points) What does isotropic mean?
    A. Different types of atomic nuclei, with the same number of protons, but different number of neutrons
    B. Same throughout time
    C. Translationally symmetric
    D. Rotationally symmetric
    E. Having the same (average) composition throughout

23. (2 points) The Chandra Isotropic Universe Survey found that the expansion of the universe was in fact, not isotropic. What is a potential explanation for this?
    A. Large groups of galaxies are moving together, and the anisotropy is due to gravity
    B. Dark energy is not uniform
    C. Both of the above
    D. Neither of the above

24. (3 points) Given that the survey relies on a relationship between temperature (which can be determined from the earth) and X-ray luminosity, then comparing this luminosity to apparent brightness, what is one source of error that might be present? Why is this unlikely to be an issue?

Use the following information for the next five (5) questions:

Image 4 shows MACS J0025.4-1222 – a galaxy cluster notable because it provides clear evidence for the existence of dark matter. (Note: This object is not on your list of required objects for 2020.)

25. (3 points) Even though MACS J0025.4-1222 is not your list of required objects in 2020, an object like it (not only physically, but in its significance) is. What is this object called?

26. (2 points) The object from the previous question is shown twice in the Image Set. List which images show this object, as well as which part of the electromagnetic spectrum they were taken in.

27. (3 points) One of the images from the previous question contains a false color highlight that does not represent any part of the electromagnetic spectrum. What, instead, does it show, and how do astronomers measure this quantity?

28. (3 points) Briefly explain what caused this object to take on its current morphology (structural properties).

29. (3 points) Building from your answer to the previous question, why does this object provide direct evidence for dark matter?

Section C: Calculations and Long-Form Questions

Answer the following calculation and long-form questions. For calculations, give a brief summary of your process for credit (and partial credit if the answer isn't correct). Calculations without explanations will be awarded minimal credit. Questions are worth 3-4 points each, for a total of 158 points.

1. (15 points) **Angular Measurements and Orbital Mechanics.** You're a part of a team of astronomers that has just discovered a new binary star system travelling around a supermassive black hole. First, you examine the binary system...

   (a) (3 points) The stars in the binary system have a mean angular separation of 1 milliarcseconds. Given that the Earth is 8 kpc from the center of the Milky Way, what is the mean separation of the binary system, in AU? (In other words, convert that angular separation to a linear distance.) Round your answer to the nearest whole number.

   (b) (3 points) Long term observations of the star system show that it has an orbital period of 8 Earth years. What is the total mass of the binary star system, in solar masses? Round your answer to the nearest whole number.

   (c) (3 points) Radial velocity measurements indicate that one star is moving at 3 times the speed of the other. What is the mass of the more massive star in this system, in solar masses? Round your answer to the nearest whole number.

   Next, you turn your focus to the supermassive black hole (SMBH). The supermassive black hole at the center of a galaxy has a mass of $4 \times 10^6\,\mathrm{M}_\odot$.

   (d) (3 points) From previous observations, you know that mean separation between the binary star system and the SMBH is 8 pc. At its periapsis (closest point in the orbit), the binary system is only half as far away. At this point, how fast is the star moving, in $\mathrm{m\,s^{-1}}$?

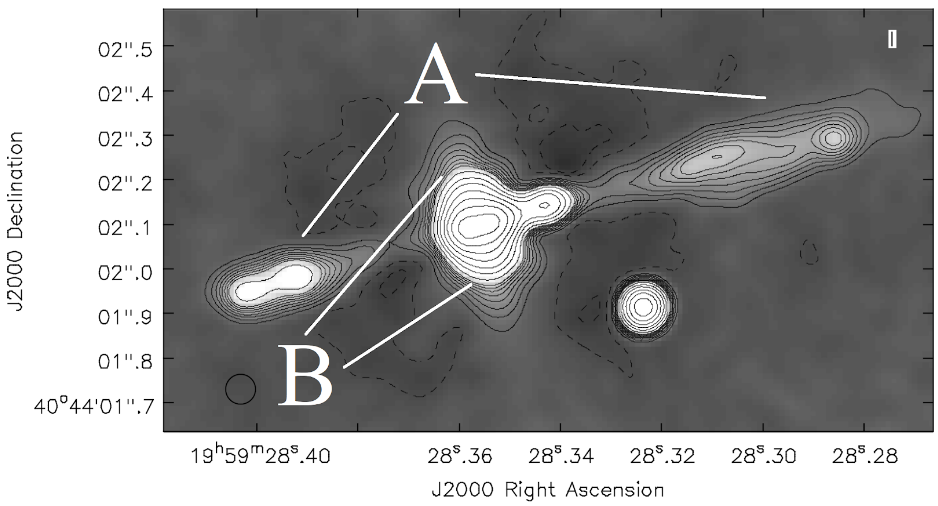
   (e) (3 points) Based on the information given in the previous part, what is the eccentricity of the orbit of the binary system around the SMBH? Round to one decimal place.

2. (15 points) **AGN / Cygnus A Radio Image Analysis.** Active galactic nuclei (AGNs) are a complicated, ongoing field of research for astronomers. Initially, when astronomers first observed AGNs, they saw a wide collection of objects with disparate properties. However, they ultimately were able to merge all these objects into the class AGN using the unified model.

   (a) (3 points) Explain how the unified model accounted for all of the AGN classes.

   (b) (3 points) Name and describe the features present in the unified model.

   Cygnus A is one of the first radio galaxies discovered by astronomers. The image below (Carilli et al. 2019) depicts a high resolution, radio observation of Cygnus A:



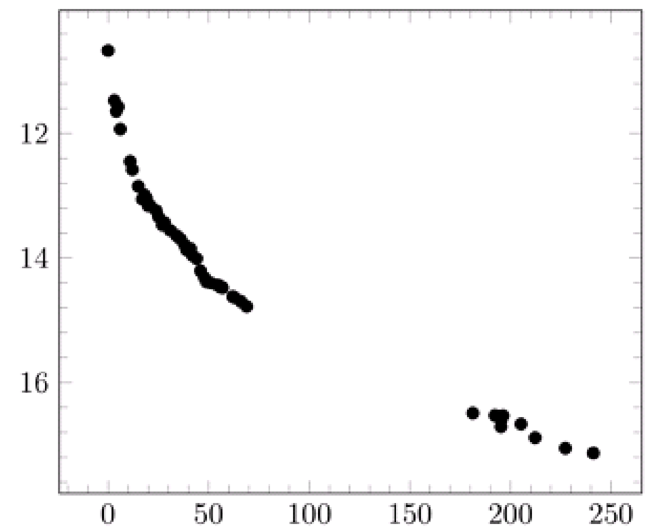
   (c) (3 points) Identify the features marked A and B.

   (d) (3 points) Feature B emits strongest in which portion of the electromagnetic spectrum?

   A. Radio
   B. Gamma-ray
   C. X-ray
   D. Optical
   E. Infrared

   (e) (3 points) Given that the distance to Cygnus A is 750 million light years and using the largest contour, estimate the length (longest side) of feature B, in pc.

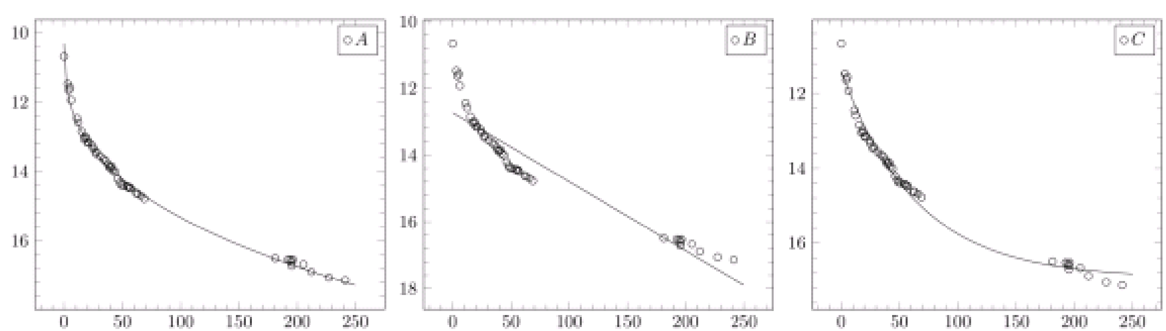
3. (9 points) **Fitting Data.** Consider the following V-band photometry of V1187 Sco (Walter et al. 2012):



You fit the data with the following three functional forms:

1. A linear regression in the form $M = at + b$
2. A power regression in the form $M = at^b + c$
3. An exponential regression in the form $M = ae^{bt} + c$

The fits are shown (in an arbitrary order) in the image below:

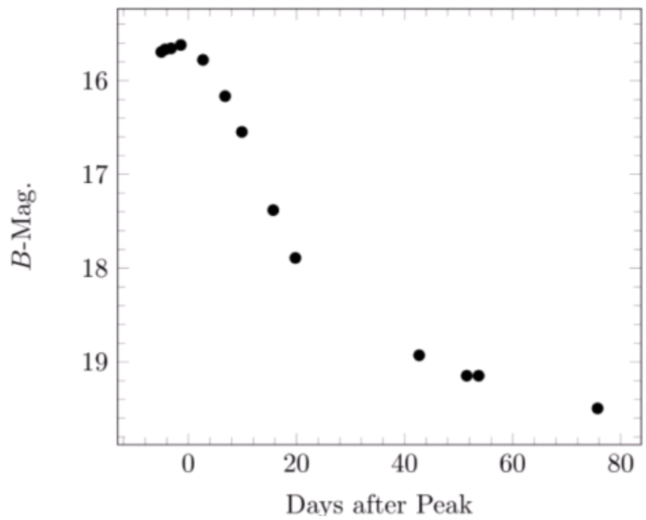


(a) (3 points) Which regression fits the data best (A-C)?

(b) (3 points) Identify the power regression and explain how you found it.

(c) (3 points) Why might this comparison between fits be "unfair"? In other words, what disadvantage does one of these regression methods have over the others?

4. (18 points) **Type Ia Supernovae.** Type Ia supernovae decays are partially powered by radioactive decay. Consider the following light curve of SN 97e:



(a) (3 points) The Phillips relation in the B, V and I bands are

$$M_B = -21.726 + 2.698 \times \Delta m_{15}(B)$$

$$M_V = -20.883 + 1.949 \times \Delta m_{15}(B)$$

$$M_V = -19.591 + 1.076 \times \Delta m_{15}(B)$$

where $\Delta m_{15}(B)$ is the decline in the B-magnitude light curve from maximum light to the magnitude 15 days after B-maximum.

If extinction **is** a factor, which of the above relations (if any) would yield the most accurate calculation for the distance?

A. B band
B. V band
C. I band
D. None of the above/other

(b) (3 points) Use the appropriate relation to calculate the distance to this event, in Mpc.

(c) (3 points) Suppose we instead assumed the absolute magnitude of the event was -19.5. Would this affect our distance measurement?

A. We'd overestimate the distance
B. We'd underestimate the distance
C. This would not impact our measurement of the distance

(d) (3 points) Ni-56 decay is a major component of Type Ia supernovae with a half life of 6.1 days. If 0.5 solar masses is produced in the event, how many nuclei does this represent? It might be useful to know 1.66×10^{-27} kg is equivalent to one atomic mass unit.

(e) (3 points) What percent of the nuclei remain after 10 days?

(f) (3 points) If each decay produces 1.75 MeV in photons, how much energy is released in this time, in Joules?

5. (16 points) **Neutron Stars.** Neutron stars are curious objects that are formed from the collapse of a massive star at the end of its life. Let's examine some of their properties.

   (a) (4 points) If a 1 kilogram mass weighs $7.5 \times 10^{12}\,\mathrm{N}$ at the surface of a neutron star, find the approximate radius of the neutron star in $\mathrm{R}_\odot$. You may assume that its density is uniform and of order $10^{17}\,\mathrm{kg\,m^{-3}}$.

   (b) (4 points) If the neutron star was in hydrostatic equilibrium, find to order of magnitude the pressure at its center in Pa.

   (c) (4 points) For all neutron stars, there is a maximum mass above which the neutron star is not stable. Briefly explain why this is.

   (d) (4 points) Many neutron stars, once born, are rapidly spinning. Explain why rotation increases the upper mass bound for neutron star stability.

6. (16 points) **Black Holes.** Black holes are perhaps even more curious objects. Different types of black holes exist, but stellar-mass black holes are formed from the collapse of an extremely massive star, usually over $8 - 10\,\mathrm{M}_\odot$. Much research has been devoted to understanding the mysterious properties of these objects.

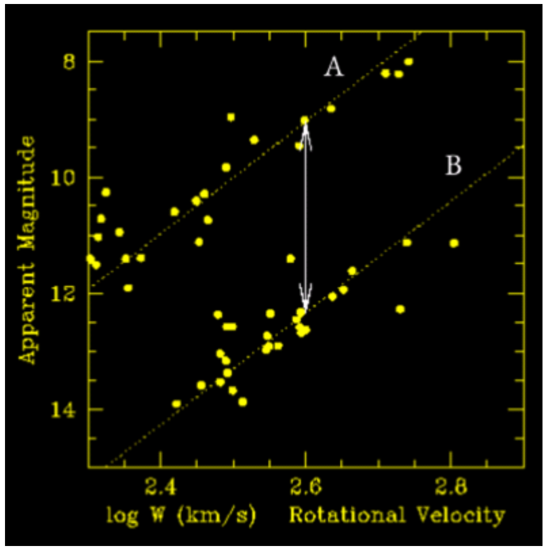
   (a) (4 points) Two commonly used conjectures about black holes are the cosmic censorship hypothesis and the "no hair" theorem. Describe what each of these conjectures say about black holes.

   (b) (4 points) It is thought that extremely small, so-called Planck particles or planckions were present (very) shortly after the Big Bang. These particles are thought to be the smallest black holes possible. Use fundamental constants to derive an order of magnitude estimate for the mass of a planckion in kg.

   (c) (4 points) It turns out that the cosmic censorship hypothesis necessitates an upper bound for the angular momentum of a black hole. Modeling a rotating black hole as a disk, approximate this maximum angular momentum for a solar mass black hole in $\mathrm{kg\,m^2\,s^{-1}}$.

   (d) (4 points) Singularities are the innermost parts of black holes where the laws of physics are thought to break down. Furthermore, some astrophysicists believe that it is possible for singularities not enclosed by event horizons to occur, resulting in so-called "naked" singularities. Describe the shape a singularity would take for a rotating black hole, and propose an explanation for how "naked singularities" might emerge.

7. (28 points) **Galaxy Distances.** Shown below is a graph of apparent magnitude vs spectral line width rotational velocity for two different clusters of spiral galaxies, Cluster A and Cluster B. (Image Credit: NOAO)



(a) (4 points) What is the relationship between these two properties called?
   A. Kormendy relation
   B. Tully-Fisher relation
   C. Faber-Jackson relation
   D. M-sigma relation

(b) (4 points) Which galaxy cluster is farther away?
   A. Cluster A
   B. Cluster B
   C. Impossible to determine

Astronomers have measured the H-α emission line ($lambda_0 = 656.281\,\mathrm{nm}$) for a particular galaxy on this diagram to be centered at 662.406 nm with a width of 1.202 nm. This galaxy has an apparent magnitude of 11.1.

(c) (4 points) What is the distance to this galaxy, in Mpc? Use $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

(d) (4 points) What is the quantity $\log W$ for this galaxy, where W is in $\mathrm{km\,s^{-1}}$?

(e) (4 points) Is this galaxy in Cluster A or Cluster B?

(f) (4 points) What is the distance to Cluster A? Cluster B? Give your answers in Mpc.

(g) (4 points) This power law relationship between spectral line width (observable property) and magnitude makes galaxies and galaxy clusters a significant rung of the cosmic distance ladder. This empirical relationship can be parameterized as

$$M = \alpha + \beta \log W$$

where M is the absolute magnitude of the galaxy and W is the line width velocity in $\mathrm{km\,s^{-1}}$. Using the distances found in the previous question and the graph, find the values of the parameters α and β. Assume there is no interstellar extinction.

8. (40 points) **Expansion of the Universe.** *Note: Parts (g) and (h) involve the use of basic differentiation.*

In this problem, you will explore a uniformly expanding universe. Let us have a comoving coordinate system that expands along with the universe, such that two galaxies A and B moving away from each other with the expansion of the universe will have a fixed comoving distance, while the proper distance between the two (i.e. "actual" distance) increases with time. The two are related by the scale factor $a(t) = l_p/l_c$ which describes the relative size of the universe as a function of time. A common way to define distances in our comoving coordinate system is to define $a(t_0) = 1$, where t_0 refers to the present. In other words, we define the comoving distance to a galaxy to be the proper distance to that object in the present. With this convention, $a(t)$ is simply the size of the universe relative to what it is today.

Suppose that Galaxy A is currently 500 Mpc from us. Using this information, answer the next four (4) questions.

(a) (4 points) What is the comoving distance to Galaxy A right now, in Mpc?

(b) (4 points) What is the proper distance to Galaxy A right now, in Mpc?

(c) (4 points) What is the comoving distance to Galaxy A when $a(t) = 2$, in Mpc?

(d) (4 points) What is the proper distance to Galaxy A when $a(t) = 2$, in Mpc?

(e) (4 points) Just as distances stretch with the expansion of the universe, so do wavelengths. If a galaxy moving with the Hubble flow emits a photon at $t = t_e$, derive an expression for its redshift z in terms of $a(t_e)$, which is the scale factor at the time light left the galaxy. *Hint:* $1+z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{\lambda(t_0)}{\lambda(t_e)}$

(f) (4 points) The most distant galaxy discovered to date is GN-z11, which, as its name implies, has a redshift of $z = 11.09$. How many times smaller was the universe at the time light left GN-z11? (This number should be greater than 1.)

(g) (4 points) It is possible to use the definition of the scale factor to show that for a uniformly expanding universe described by the scale factor $a(t)$, velocity is proportional to proper distance ($dl_p/dt = H(t)l_p$), where the Hubble parameter $H(t)$ is the constant of proportionality. Hubble's law is therefore proof that we live in a uniformly expanding universe. Find $H(t)$ in terms of $a(t)$. (Recall that the comoving distance l_c is a constant with respect to time.)

(h) (4 points) In a matter-dominated homogeneous and isotropic universe, the scale factor is given by $a(t) = bt^{2/3}$, where t is the time since the Big Bang and b is an arbitrary constant. (This can be found by integrating the Friedmann equation.) Using the expression derived in the previous question, find an expression for the age of such a universe in terms of the Hubble constant, H. The constant b should not be in your answer.

(i) (4 points) What would the age of our universe be if it was a matter-dominated, homogeneous, and isotropic universe, in Gyr? Assume $H(t_0)$ is $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Be careful of units. Is this consistent with the fact that the oldest stars found in the Milky Way halo have an age of 13 Gyr?

(j) (4 points) Is our universe matter-dominated? If not, what mass-energy does dominate our universe?