Astronomy C - Astronomy - Final - MIT Invitational - 01-22-2021

Welcome to the MIT Invitational 2020! This test is for Astronomy in Division C. You will have 50 minutes to complete this exam, and you **are** allowed to use the internet. All images for this exam are in a PDF as a "Supporting Document" in Scilympiad for this exam. If you run into trouble accessing the image sheet through Scilympiad, you can also find it here (https://tinyurl.com/mitastro21) [https://tinyurl.com/mitastro21]. We recommend downloading the image sheet for better viewing.

This exam consists of 125 questions for a total of 215 points. Generally, the questions are ordered by difficulty, with the easiest questions at the start. However, there are numerous exceptions, so we strongly recommend looking at every question and taking the test in a way that plays to your strengths. Furthermore, we recommend answering every question you can, even if you're not sure about your answer, as there's no penalty for guessing!

Astronomy is a unique event in that it incorporates aspects of science that aren't often covered in a normal class in school. We've done our best to show what a beautiful and diverse subject it is through the questions on this exam, even with the limitations a virtual tournament places on us. Above all else, just believe!

Section A

1. (1.00 pts) What type of galaxy is the Milky Way?

- ○ A) Spiral
- ◉ B) Barred-spiral
- ○ C) Elliptical
- ○ D) Irregular

2. (1.00 pts) Astronomers suspect that a ____________ lies at the center of almost every galaxy

- ○ A) Quasar
- ◉ B) Supermassive black hole
- ○ C) Neutron star
- ○ D) AGN

3. (1.00 pts) Which of the following correctly lists the standard Harvard spectral classification scheme in order of increasing temperature?

- ○ A) OBAFGKM
- ◉ B) MKGFABO
- ○ C) OFGKAMB
- ○ D) MFGKOBA

4. (1.00 pts) Which Harvard spectral class is characterized by the presence of many absorption lines from molecules and neutral metals?

- ○ A) O
- ○ B) A
- ○ C) G
- ◉ D) M

5. (1.00 pts) Which of the following best explains your answer to the previous question?

- ◉ A) Cooler temperatures allow molecules on/near the surface of the star to stay intact. Compounds have molecular orbitals that allow more complicated electronic transitions than a single atom.
- ○ B) Higher temperatures mean that atoms near the surface move at higher speeds, which in turn gives them more variability in the radiation they absorb.
- ○ C) Lower mass stars have higher surface gravities, meaning that surrounding gas will be held more tightly, increasing the probability of an atom absorbing radiation from the star

- D) Higher temperatures increase the probability of having atoms in the star ionized, which makes those atoms more susceptible to absorbing radiation at a later time.

6. (1.00 pts) Why are hydrogen (Balmer) lines weak in the coolest classes of stars?

- A) Cooler stars have a significantly higher metallicity, causing hydrogen absorption lines to be weaker.
- B) Temperatures in cooler stars are not high enough, and thus most of the hydrogen atoms are in the ground state. (selected)
- C) Hydrogen is in an ionized plasma state in these types of stars, and ionized hydrogen atoms do not produce absorption lines.
- D) Cooler stars are powered by the fusion of deuterium, which has different spectral lines than the normal isotope.

7. (1.00 pts) The average distance between the Earth and the Sun is

- A) 1 AU (selected)
- B) 1 GU
- C) 1 pc
- D) 1 ly

8. (1.00 pts) Which of the following is closest to the number of lightyears in a parsec?

- A) 3.26 (selected)
- B) 3.62
- C) 180
- D) 206265

9. (1.00 pts) Which of the following comprises the most mass-energy of the current universe?

- A) Dark matter
- B) Dark energy (selected)
- C) Baryonic matter
- D) Electromagnetic radiation

10. (1.00 pts) Which of the following comprises the most mass-energy of the **early** universe ($t \lesssim 50000$ years)?

- A) Dark matter
- B) Dark energy
- C) Baryonic matter
- D) Electromagnetic radiation (selected)

11. (1.00 pts) Which of the following is closest to the radius of the Earth in solar radii?

- A) 0.0009
- B) 0.009 (selected)
- C) 0.09
- D) 0.9

12. (1.00 pts) The parallax of a distant star is measured as 2.3 milliarcseconds. Find its distance in parsecs.

- A) 217

- B) 435
- C) 870
- D) 1420

13. (1.00 pts) The main sequence is characterized predominantly by the fusion of which of the following elements?

- A) Carbon
- B) Nitrogen
- C) Helium
- D) Hydrogen

14. (1.00 pts) Currently, nuclear fusion in the Sun occurs via the

- A) Proton-proton chain
- B) CNO cycle
- C) Triple alpha process
- D) Saha process

15. (1.00 pts) Which of the following is a satellite galaxy of the Milky Way?

- A) LMC
- B) M31
- C) Andromeda Galaxy
- D) M101

16. (1.00 pts) Which of the following is closest to the Chandrasekhar limit in solar masses?

- A) 0.72
- B) 1.0
- C) 1.4
- D) 2.2

17. (1.00 pts) The Tolman–Oppenheimer–Volkoff limit applies to

- A) Galaxies
- B) Globular clusters
- C) White dwarfs
- D) Neutron stars

18. (1.00 pts) Ignoring inconsistencies due to double degenerate models, what is the approximate peak absolute magnitude of a Type Ia supernova?

- A) 19.3
- B) 9.5
- C) -9.5
- D) -19.3

19. (1.00 pts) Which acronym refers to the gravitational wave detector with bases in Washington State and Louisiana?

- A) LISA
- B) JWST
- C) LIGO
- D) VLBA

20. (1.00 pts) Which of the following spectral lines/features has been used prominently to probe the kinematics of the Milky Way (i.e. rotation of its spiral arms)?

- A) 21 cm line
- B) Hydrogen α
- C) Calcium H and K lines
- D) Lyman α forest

21. (1.00 pts) A distant galaxy has an angular size (diameter) of 20 arcminutes. Additional observations show that it is 6 Mpc away. What is the diameter of this galaxy, in kpc?

- A) 0.58 kpc
- B) 1.74 kpc
- C) 34.09 kpc
- D) 2000 kpc

22. (1.00 pts) A distant RR Lyrae variable is observed to have apparent magnitude 6.25. Find its approximate distance in pc.

- A) 10.1 pc
- B) 126 pc
- C) 2540 pc
- D) 12600 pc

23. (1.00 pts) Using information from the previous question, correct this distance estimate assuming total extinction due to intervening dust was 2.41 magnitudes.

- A) 2.56 pc
- B) 41.5 pc
- C) 178 pc
- D) 512 pc

24. (1.00 pts)
Aspiring astronomer Chika Fujiwara calculated the distance to the Small Magellanic Cloud by applying Hubble's law to its redshift $z = 0.000527$, and claims that the SMC is 4.2 Mpc away. Why is she wrong? Select all that apply.

(Mark **ALL** correct answers)

- A) The expansion of the universe only occurs at large distances ($\gtrsim 100$ Mpc).
- B)
Since the Small Magellanic Cloud is in the Local Group, its motion relative to us is influenced more by gravitational interactions with the Local Group than the expansion of the universe.
- C) She did the calculation wrong.

25. (1.00 pts) Select all that apply: Stars with a higher metallicity look redder because

(Mark **ALL** correct answers)

- A) Many metals, such as iron, have more absorption lines in the bluer end of the spectrum
- B) The presence of metals increases the star's opacity, causing it to be larger and cooler

- [ ] C) It takes more energy to heat metals than hydrogen gas; this causes the star to be cooler and redder

26. (1.00 pts) Which of the following is not true of the "Rayleigh–Jeans catastrophe"?

- ○ A) It arises from not taking quantum mechanics into account.
- ○ B) A graph of the Jeans black body radiation curve diverges as λ approaches 0.
- ○ C) It predicts that infinitely high frequencies can be excited, since there is no upper limit.
- ● D) It predicts that a region of space containing an electromagnetic field will never come into thermal equilibrium, since the electromagnetic field will absorb energy infinitely.

27. (1.00 pts) Besides its effective temperature, the black body radiation curve (spectral radiance) of an object depends on which of the following?

- ● A) Radius
- ○ B) Mass
- ○ C) Luminosity
- ○ D) None of the above

28. (1.00 pts) The wavelength at which the spectrum of a blackbody peaks is given by

- ○ A) the Stefan-Boltzmann Law
- ○ B) Hubble's Law
- ● C) Wein's Law
- ○ D) the Tully-Fisher relation

The following three (3) questions refer to Image A.

29. (1.00 pts) Which location on the HR Diagram represents the Sun?

- ○ A) H
- ● B) F
- ○ C) A
- ○ D) M

30. (1.00 pts) Which location on the HR Diagram represents white dwarfs?

- ● A) A
- ○ B) B
- ○ C) S
- ○ D) P

31. (1.00 pts) Of the following locations on the HR Diagram, which corresponds to the star with the shortest main-sequence lifetime?

- ○ A) M
- ○ B) B
- ○ C) R
- ● D) H

The following five (5) questions refer to Image B:

32. (1.00 pts) This HR diagram plots the stars of 32 different

- ○ A) Starburst galaxies
- ○ B) Elliptical galaxies
- ◉ C) Open clusters
- ○ D) Globular clusters

33. (1.00 pts) The color on the graph reflects each object's

- ○ A) Temperature
- ○ B) Mass
- ◉ C) Age
- ○ D) Radius

34. (1.00 pts) Which location on the HR Diagram represents stars that are undergoing a helium flash?

- ○ A) A
- ○ B) B
- ◉ C) C
- ○ D) D
- ○ E) E
- ○ F) None of the above

35. (1.00 pts) Which location on the HR Diagram represents stars that are on the subgiant branch?

- ○ A) A
- ○ B) B
- ○ C) C
- ○ D) D
- ○ E) E
- ◉ F) None of the above

36. (1.00 pts) Which location on the HR Diagram represents stars that are on the red giant branch?

- ○ A) A
- ◉ B) B
- ○ C) C
- ○ D) D
- ○ E) E
- ○ F) None of the above

37. (2.00 pts) Order Images 15, 16, and 17 by the physical size of the DSOs they depict, from largest to smallest.

- ○ A) 15, 16, 17
- ◉ B) 15, 17, 16
- ○ C) 16, 15, 17
- ○ D) 16, 17, 15

- E) 17, 15, 16
- F) 17, 16, 15

38. (1.00 pts) Of the images in the previous question, which shows the DSO that is farthest from Earth?

- (selected) A) Image 15
- B) Image 16
- C) Image 17

The following three (3) questions refer to Image 7 and Image 8:

39. (1.00 pts) Which DSO is shown in Image 7?

- (selected) A) JKCS 041
- B) M87
- C) 3C 273
- D) GW151226

40. (1.00 pts) This DSO is scientifically significant because it is/was

- A) The site of the first gravitational wave event due to neutron stars
- B) The oldest globular cluster in the universe
- (selected) C) The most distant galaxy cluster at the time of its discovery
- D) The most distant non-quasar galaxy

41. (2.00 pts) Image 8 shows an image of this DSO taken by the Hubble Space Telescope. The lack of a single dominant object at the center is best explained by

- (selected) A) This DSO's relative youth
- B) All the objects in this DSO being approximately the same mass
- C) Collisions with other clusters that result in tidal disruptions, shifting objects inside the DSO around frequently
- D) The wide range of ages of the objects in the DSO (i.e. some objects are very old, while some are very young)

The following four (4) questions refer to the DSO pictured in Image 9:

42. (1.00 pts) Which DSO is shown in Image 9?

- A) DLA0817g
- B) MACS J1149.5+2223
- C) PSS 0133+0400
- (selected) D) 3C 273

43. (1.00 pts) Which telescope took this image?

- A) Chandra
- (selected) B) Hubble
- C) Fermi
- D) Spitzer

44. (1.00 pts) Which image shows the constellation in which this DSO resides?

- ○ A) Image 1
- ○ B) Image 4
- ◉ C) Image 6
- ○ D) Image 13

45. (2.00 pts) The mass of this DSO's central black hole was measured using which of the following techniques?

- ◉ A) Reverberation mapping
- ○ B) Gravitational lensing
- ○ C) M-sigma relation
- ○ D) Radial velocity measurements

The following five (5) questions refer to PSS 0133+0400:

46. (1.00 pts) Which image shows PSS 0133+0400?

- ○ A) Image 2
- ◉ B) Image 5
- ○ C) Image 12
- ○ D) Image 14

47. (1.00 pts) In which portion of the electromagnetic spectrum was the image from the previous question taken?

- ○ A) Visible
- ○ B) Infrared
- ◉ C) X-ray
- ○ D) Radio

48. (1.00 pts) Which image shows the constellation in which this DSO resides?

- ○ A) Image 1
- ○ B) Image 4
- ○ C) Image 6
- ◉ D) Image 13

49. (1.00 pts) Which of the following would be an appropriate technique to determine the distance to this DSO?

- ◉ A) Hubble's Law
- ○ B) Gravitational lensing
- ○ C) Type Ia supernovae
- ○ D) None of the above

50. (1.00 pts) The x-ray emission spectrum of this DSO involves which of the following spectral features?

- [ ] A) Absorption line
- [ ] B) Emission line
- [ ] C) Blackbody radiation
- [x] D) None of the above

The following three (3) questions refer to Image 14:

51. (1.00 pts) In which portion of the electromagnetic spectrum was Image 14 taken?

- [ ] A) Visible
- [ ] B) Infrared
- [ ] C) X-ray
- [x] D) Radio

52. (1.00 pts) Which telescope collected the data used to create this image?

- [ ] A) Hubble
- [ ] B) Spitzer
- [ ] C) Chandra
- [x] D) VLA
- [ ] E) Arecibo

53. (2.00 pts) Select all that apply: The structures visible in Image 14 are

(Mark **ALL** correct answers)

- [x] A) Shock waves
- [ ] B) Expelled material from post-main sequence stars
- [x] C) Turbulent gas flow
- [ ] D) The edges of a nebula

Astronomers have discovered quasars from when the universe was less than a billion years old, but there isn't a consensus on how such large black holes were able to form so quickly. GOODS-S 29323 offers some of the best evidence to date in support of so-called "direct collapse black holes" being a likely formation mechanism. **The following four (4) questions refer to this DSO.**

54. (1.00 pts) There is one image that shows GOODS-S 29323 on the image sheet. In which portion of the electromagnetic spectrum was it taken?

- [ ] A) Visible
- [ ] B) Infrared
- [x] C) X-ray
- [ ] D) Radio

55. (2.00 pts)
One theory to explain these early supermassive black holes is that seed black holes formed shortly after the Big Bang would accrete gas until they reached the size of supermassive black holes. Which of the following choices is a weakness of this theory?

- [ ] A) The number of seed black holes immediately after the start of the Big Bang may not be large enough to account for the number of distant quasars we observe today.
- [x] B) The rate at which seed black holes accrete gas and dust would be too low to explain their current sizes
- [ ] C) Seed black holes are likely to evaporate due to Hawking radiation before becoming large enough, as they have extremely low initial masses
- [ ] D) None of the above

56. (2.00 pts)
In the formation of a direct collapse black hole, an extremely large gas cloud collapses without fragmenting into a black hole. Which of the following elements would you expect to see in the gas cloud? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) Hydrogen
- [x] B) Helium
- [ ] C) Lithium
- [ ] D) Carbon
- [ ] E) Iron

57. (2.00 pts)
When astronomers are searching for direct collapse black hole candidates (like GOODS-S 29323), they filter out objects that do not show strong x-ray emissions. Why would this be the case?

- ○ A) The process of the gas cloud collapsing releases a lot of gravitational potential energy, which would be emitted in the form of x-rays.
- ○ B) Leftover radiation from the Big Bang results in matter-photon interactions, which in turn release copious amounts of x-rays.
- ◉ C) Nearby supermassive black holes exhibit strong x-ray signals due to their accretion disks, and astronomers suspect distant black holes would be the same.
- ○ D) None of the above

The following four (4) questions refer to NGC 2623, an interacting galaxy in the constellation of Cancer.

58. (2.00 pts)
Suppose you take a multi-wavelength image of the galaxy NGC 2623 and notice strong UV and IR emission compared to X-ray emission. Which of the following reasons would best explain this finding?

- ◉ A) Active star formation regions due to starburst
- ○ B) High metallicities and dust content
- ○ C) Abnormally high supernova frequency
- ○ D) Extremely active supermassive black hole

59. (1.00 pts) Which of the following telescopes would be most appropriate for taking an IR image of NGC 2623?

- ◉ A) Spitzer
- ○ B) GALEX
- ○ C) Hubble
- ○ D) VLA/VLBA

60. (2.00 pts)
Suppose that the average mass of a main sequence star in NGC 2623 is 4.5 solar masses. What is the ratio of the expected lifetime for stars in this galaxy to that of the Sun?

- ○ A) 0.0023
- ◉ B) 0.023
- ○ C) 0.23
- ○ D) 2.3

61. (2.00 pts)
In 2008, Evans et al. reported on star formation activity in NGC 2623, citing the presence of [Ne V] 14.3 μm emission. Which of the following best explains the presence of this spectral line?

- ◯ A) Dusty sites in the spiral arms are ideal for the existence of heavy molecules and ions.
- ◯ B) Supernova nucleosynthesis has resulted in an abundance of Ne and post-iron-peak elements.
- ◉ C) The AGN is responsible for the ionization of many neutral elements.
- ◯ D) None of the above

The following seven (7) questions concern the Chandra Isotropic Universe Survey and the isotropy of the universe. Some of the questions will use Image 18, which show the 313 galactic clusters surveyed in the study at various redshifts in galactic coordinates.

62. (1.00 pts) The two crucial pillars of theoretical cosmology (i.e. the two fundamental assumptions upon which cosmological models are built) are isotropy and

- ◉ A) Homogeneity
- ◯ B) Dark energy
- ◯ C) Curvature
- ◯ D) Flatness

63. (2.00 pts)
By and large, the universe is remarkably isotropic, as is observed in the Cosmic Microwave Background. Why might the CMB not be sufficient evidence for the isotropy of the universe?

- ◯ A) The CMB does not show anisotropies in the universe from lower redshifts
- ◯ B) The CMB would not reflect anisotropies from dark energy
- ◯ C) The CMB is too uniform for us to make good measurements of the universe's anisotropies
- ◉ D) All of the above

64. (2.00 pts) Why are there blank spots near galactic longitude 270° in the Southern galactic hemisphere? Select all that apply.

(Mark **ALL** correct answers)

- ☐ A) Due to the uneven (anisotropic) distribution of galaxy clusters in the universe
- ☑ B) Due to obscuration from a nearby galaxy or galaxies
- ☐ C) Due to obscuration from dust in the halo of the Milky Way

65. (2.00 pts)
How did astronomers use galaxy clusters as a standard candle to determine its luminosity distance independently from any assumptions about cosmological parameters? (This is important because such a method would allow the luminosity distance measures to help fit cosmological parameters.)

- ◉ A) They calculated the X-ray luminosity of the galaxy cluster from its temperature, and then used the inverse-square law with the measured x-ray flux
- ◯ B) They calculated the distance to the galaxy cluster by using its redshift and Hubble's law, and from there calculated the luminosity distance
- ◯ C) They measured its angular diameter and velocity dispersion from the thickness of spectral lines, and used the Sigma-D relation
- ◯ D) They used Type Ia supernovae observed in the galaxy clusters as standard candles

66. (2.00 pts) The luminosity distance can also be calculated from a formula that depends on several measurable parameters. Which are they?

(Mark **ALL** correct answers)

- ☑ A) Redshift
- ☐ B) Hubble constant
- ☑ C) Proper distance
- ☑ D) Density parameters

67. (2.00 pts)

In the paper in which these results were published, the authors did a best fit for a certain parameter for all the galaxy clusters in this survey. Using these results, they were able to find that certain directions of the universe appeared to expand faster than other regions. Which parameter was this?

- ○ A) Redshift
- ◉ B) Hubble constant
- ○ C) Proper distance
- ○ D) Density parameters

68. (2.00 pts) Which of the following is a plausible explanation for the results of the survey?

- ○ A) Dark energy is distributed unevenly in the universe, which would undermine a major assumption made in many cosmological models
- ○ B) The grouping of galaxy clusters caused the clusters to not move exactly with the Hubble flow
- ○ C) Interstellar (and even intergalactic) extinction could interfere with measurements of flux and therefore calculations of luminosity distance
- ◉ D) All of the above

The following five (5) questions refer to Images 10 and 11, both of which show 1E0657−558. This DSO is especially significant in that it offered astronomers another way to prove the existence of dark matter.

69. (1.00 pts) Which of the following is another name for 1E0657−558?

- ○ A) H1821+643
- ◉ B) Bullet Cluster
- ○ C) Andromeda Galaxy
- ○ D) Pinwheel Galaxy

70. (1.00 pts) There is one other image in this set that shows 1E0657−558. Which one is it?

- ○ A) Image 2
- ◉ B) Image 3
- ○ C) Image 9
- ○ D) None of the above

71. (2.00 pts)
Typically, dark matter is observed indirectly. For most DSOs, dark matter and baryonic matter are in roughly the same location, making them difficult to tell apart. Why would the dark matter and baryonic matter in this DSO be spatially segregated?

- ○ A) During the galaxies' collision, gravitational interactions preferentially ejected dark matter compared to baryonic matter.
- ○ B) Weak interactions between clumps of dark matter cause them to coalesce in the center of the DSO, while baryonic matter is more spread out
- ◉ C) While both baryonic and dark matter interact gravitationally, baryonic matter will also experience a friction-like electromagnetic force, causing it to move more slowly
- ○ D) Collisions with antimatter in the galaxies will preferentially destroy baryonic matter, which allows the dark matter in the system to occupy a larger area

72. (1.00 pts) The blue, red, and yellow data behind the green contour in Image 11 represents the distribution of ______ in this DSO

- ○ A) Dark matter
- ◉ B) Baryonic matter

73. (1.00 pts) What technique was used to create the green contours shown in both Images 10 and 11? (Note: the contours are the same in both images)

- ○ A) Repeated parallax measurements over an extended period of time

- B) Radial velocity measurements
- C) Velocity dispersion relations
- D) Gravitational lensing

Section B

74. (2.00 pts)
As a crude estimate of the pressure due to electrons in a white dwarf or the degenerate core of a red giant, we can model the free electrons as an ideal gas. Assuming that the degenerate core we are considering has solar mass, Earth-like radius, and temperature 10^5 K, estimate the free electron pressure in Pa.

- A) 3×10^{10}
- B) 3×10^{15}
- C) 3×10^{21}
- D) 3×10^{28}
- E) 3×10^{35}

75. (2.00 pts) Which of the following is the most accurate explanations for why Balmer lines are weaker for hotter stars?

- A) More rapid hydrogen fusion decreases neutral hydrogen abundance
- B) More rapid hydrogen ionization
- C) High metallicities wash out neutral hydrogen lines
- D) The stronger Lyman series overwhelms the Balmer lines

76. (2.00 pts) Which of the following conditions is necessary for the application of the Saha equation?

- A) Thermodynamic equilibrium
- B) High gas densities
- C) High gas temperatures
- D) High gas pressures

77. (2.00 pts) At which of the following temperatures would the strength of Mg II spectral lines be strongest?

- A) 13,000 K
- B) 10,000 K
- C) 7,000 K
- D) 4,000 K

78. (2.00 pts) Given the stress-energy tensor for dark matter, what is the ratio of its pressure to its energy density?

- A) -2
- B) -1
- C) 0
- D) 1/3

79. (2.00 pts)
For isotropic blackbody radiation (a fair approximation for the photon gas within stars), by what factor does the total radiation pressure change if we double the temperature?

- A) 1/2

- B) $\sqrt{2}$
- C) 2
- D) 16

80. (2.00 pts)
Which of the following spectral features is observed in stars (including the Sun) as a result of greater opacity (and correspondingly lower brightness) for wavelengths less than about 365 nm?

- A) Balmer jump
- B) Lyman alpha forest
- C) Schechter drop
- D) Paschen gap

81. (2.00 pts) The Rosseland mean opacity is one of several measures of opacity. How is it computed from the wavelength-dependent opacity?

- A) Quadratic mean
- B) Geometric mean
- C) Harmonic mean
- D) Arithmetic mean

82. (2.00 pts)
We normally assume main sequence stars to be in hydrostatic equilibrium. Using this and further assuming that the gravitational acceleration and density are constant throughout the star, estimate the core pressure of the Sun in Pa. Take the surface pressure to be zero. Here we're idealizing the Sun as perfectly uniform and spherically symmetric.

- A) 2.7×10^{12}
- B) 2.7×10^{14}
- C) 2.7×10^{16}
- D) 2.7×10^{18}

83. (2.00 pts) Which of the following best explains why the answer to the previous question is roughly two orders of magnitude off from the accepted value of the core pressure?

- A) Density gradient
- B) Relativistic effects
- C) Degeneracy pressure
- D) Adiabatic variations

84. (2.00 pts)
Dynamical friction is an important process in galactic interactions. If a massive object passes through a galactic halo at speed v, the force of dynamical friction on the surrounding medium is F. What is the resulting force when the speed of the object is doubled?

- A) $F/4$
- B) $F/2$
- C) $2F$
- D) $4F$

85. (2.00 pts)
The ELS collapse model suggests that the Milky Way formed from collapse of a protogalactic cloud. Estimate the timescale for this collapse if the cloud initially had mass 6.5×10^{12} solar masses and radius 45 kpc. All answers below are in years.

- A) 6×10^{3}

B) 6×10^5

C) 6×10^7

D) 6×10^9

86. (2.00 pts) Consider an ideal disk-shaped galaxy. At some reference radius r_e the surface brightness is I_e. Use de Vaucouleurs law to find the surface brightness at radius $2r_e$.

A) $0.117 I_e$

B) $0.234 I_e$

C) $0.468 I_e$

D) de Vaucouleurs law is not applicable in this scenario

87. (2.00 pts)
Consider a spherically symmetric galaxy. We know that the effect of dark matter is to flatten out the rotation curve. What dependence on the radial coordinate r must the total mass density (dark and luminous) have to yield a flat rotation curve (at sufficiently large radii)?

A) r^{-2}

B) r^{-1}

C) r^{0}

D) r^{1}

88. (2.00 pts) For which of the following types of supernovae would the problem of interstellar extinction be least severe?

A) Type Ia

B) Type Ib

C) Type Ic

D) Type II

89. (2.00 pts)
Suppose that we measure the light curve of a Type Ia supernova and find that it takes about 20 days to reach peak brightness. 22 days after this point, it can be modeled approximately by a blackbody of temperature 6200 K. If we measure the supernova's expansion velocity to be 8500 km/s, compute its absolute bolometric magnitude when it is 42 days old.

A) -17.4

B) -18.7

C) -19.4

D) -20.1

90. (2.00 pts) Which of the following types of stars would most likely be found furthest from the Galactic plane? Note that Z denotes metallicity.

A) $Z = 0.001$ stars in the thick disk

B) $Z = 0.001$ stars in the old thin disk

C) $Z = 0.05$ stars in the young thin disk

D) $Z = 0.05$ stars in the old thin disk

91. (2.00 pts)
In extragalactic astrophysics, we tend to use a bunch of different distance measures. Find the linear separation of two distant galaxies at redshift $z = 1.25$ if their luminosity distance is 3000 Mpc and their angular separation is 0.25 mas.

A) 15,000 AU

B) 0.718 pc

- C) 150 pc
- D) 2.64 kpc

92. (2.00 pts) Which of the following best explains the reason for the narrower spectral lines seen in higher-ranked stars in the Morgan-Keenan classification scheme?

- A) Higher surface gravity
- B) Lower metallicity
- C) Lower gas number densities
- D) Higher temperatures

93. (2.00 pts)
Suppose that all spiral galaxies have rotation curves that flatten out at rotation velocity v (which may be different for different galaxies). Assuming further that all spirals have the same mass-to-luminosity ratio and central surface brightness, find the dependence of luminosity on v.

- A) v^{-2}
- B) v^0
- C) v^2
- D) v^4

94. (2.00 pts) Which statement about proper distance and coordinate (comoving) distances is correct? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) The proper distance between two galaxies changes with the expansion of the universe, while the comoving distance does not.
- [ ] B) The comoving distance between two galaxies changes with the expansion of the universe, while the proper distance does not.
- [ ] C) Proper distance is measured in arbitrary units, while comoving distance is measured in units of physical distance such as meters or parsecs.

95. (2.00 pts)
The scale factor $a(t) = l_p/l_c$ is the ratio between proper and comoving distances, and describes the size of the universe over time. Use this and Hubble's law to find an expression for the Hubble parameter in terms of the scale factor.

- A) $H(t) = \frac{1}{a(t)}\frac{da}{dt}$
- B) $H(t) = a(t)\frac{da}{dt}$
- C) $H(t) = a(t)$
- D) $H(t) = \frac{a(t)}{t}$

96. (2.00 pts)
The distance modulus formula can be written as $d = 10^{\alpha(m-M)+\beta}$. Under the inverse square law and the requirement that $d = 10$ pc for $m - M = 0$, $\alpha = \frac{1}{5}$ and $\beta = 1$. In an alternate universe, light does not follow the inverse square law but instead follows an inverse cube law. Find α for this alternate universe.

- A) $1/5$
- B) $1/10$
- C) $3/10$
- D) $2/15$

97. (2.00 pts)
The luminosity distance (d_L) to a standard candle is the distance obtained from applying the inverse square law $F = \frac{L}{4\pi d_L^2}$ or $d_L = \sqrt{\frac{L}{4\pi F}}$. In a flat universe, it is related to the corresponding proper distance d_p via $d_L = (1+z)d_p$, where z is the redshift of the object. Why? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) Time dilation due to special relativity of the receding object makes its photons arrive slower, reducing the received flux by a factor of $1+z$.

- [x] B) Redshift shifts the incoming photons to lower energies, reducing the received flux by a factor of $1+z$.
- [ ] C) Redshift shifts the incoming photons to higher energies, increasing the received flux by a factor of $1+z$.

In 1998, it was discovered that the expansion of the universe was accelerating, rather than slowing down. For the following questions, refer to Image 19, which was taken from the paper publishing these results (Riess et. al 1988). It plots the distance modulus (luminosity distance) of type 1a supernovae against their redshift, and compares the results to the predictions of several models. Here, Ω is the density parameter, which describes the contributions from different types of matter to the mass-energy (and hence curvature) of the universe.

98. (2.00 pts) Which claim is best supported by the data?

- ( ) A) The universe is flat (or nearly flat) and matter dominated.
- (•) B) The universe is flat (or nearly flat) and dark energy dominated.
- ( ) C) The universe is open and matter dominated.
- ( ) D) The universe is open and dark energy dominated.

99. (2.00 pts) Which of the following are **plausible** explanations of the data in the graph? Select all that apply.

(Mark **ALL** correct answers)

- [x] A) Faraway supernovae had lower redshift than expected for their brightness; the universe was expanding slower than expected in the past, implying accelerating expansion.
- [x] B) Faraway supernovae were dimmer than expected for their redshift; early galaxies were dustier and dimmed the light from the supernovae as it travelled towards us.
- [x] C) Faraway supernovae were dimmer than expected for their redshift; the lower metal content in early white dwarfs caused dimmer supernovae.

100. (2.00 pts)
Calculate the present total density of the universe for the open-universe model. The density parameter Ω is the ratio of the density to the critical density $\rho_c = \frac{3H^2}{8\pi G}$. Use $H_0 = 67$ km/s/Mpc, where H_0 is the present-day Hubble parameter.

- (•) A) 1.7×10^{-27} kg/m^3
- ( ) B) 3.4×10^{-27} kg/m^3
- ( ) C) 6.7×10^{-27} kg/m^3
- ( ) D) 8.4×10^{-27} kg/m^3

Section C

Stellar Energy Sources. Stars produce an incredible amount of energy, and although we now know it's through nuclear fusion, finding out wasn't trivial for astronomers of the past. In this question, we'll examine why other potential energy sources aren't viable.

101. (2.00 pts)
On Earth, we produce energy mainly through chemical reactions, whether it's by burning fossil fuels (yikes) or metabolizing sugars in our body. Suppose that a "typical" chemical reaction releases 5×10^8 Joules per kilogram of fuel. What fraction of the fuel's rest mass is released as energy?

Suppose that your answer is in the form $A \times 10^B$, where A and B are both integers. Enter your answers for A and B in the first and second blanks, respectively.

[6] [-9]

102. (2.00 pts)
The Sun has a luminosity of 3.8×10^{26} Watts. Generously, let's assume that all of the Sun's mass can be used as fuel. Using your answer from the previous question, for how many years could the Sun shine at its current luminosity? *Hint: in this scenario, once fuel has been reacted, it can't be used again.*

Suppose that your answer is in the form $A \times 10^B$, where A and B are both integers. Enter your answers for A and B in the first and second blanks, respectively.

[8] [5]

103. (2.00 pts)
Another potential energy source for stars is the release of gravitational potential energy. As a rough estimate, let's assume that the Sun was formed from a molecular cloud fragment with number density 2×10^{10} H_2 molecules/m^3. If this molecular cloud fragment has the same mass as the Sun, what is its radius, in light years?

Suppose that your answer is in the form $A \times 10^B$, where A and B are both integers. Enter your answers for A and B in the first and second blanks, respectively.

2

-1

104. (2.00 pts)
Assuming that 100% of the energy was released in the collapse of the molecular cloud fragment to form the Sun went towards the Sun's luminosity, how long could the Sun shine for, in years? *Hint: don't forget about the Virial Theorem!*

Suppose that your answer is in the form 10^B, where B is an integer. Enter this integer in the blank below.

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Nuclear Fusion. As we saw in the previous set of questions, chemical reactions and the release of gravitational potential energy are not sufficient to power the Sun. Here, we'll examine the viability of nuclear fusion.

105. (3.00 pts)
Consider the nuclear reaction $4\,\mathrm{H} \longrightarrow \mathrm{He}$, where the mass of any other products (e.g. neutrinos) is negligible. Since we're trying to see if this is even *feasible* on an order-of-magnitude estimate level, let's allow the entire mass of the Sun to be used (in reality, we know that only the inner ~10% would be usable). Given that the mass of a single hydrogen atom is 1.00782503214u and the mass of a single helium atom is 4.002603u, for how many years could the Sun shine? Show your work; don't just copy and paste a tabulated value.

Expected Answer: 10^{11} years.

106. (3.00 pts)
We also have to make sure that nuclear fusion doesn't violate any conservation laws. In the first step of the proton-proton chain, two hydrogen nuclei are combined to form deuterium, while also releasing a neutrino and a positron. The positron and a free electron annihilate themselves, which releases gamma radiation. However, the neutron has more mass than the proton it used to be, and this part of the proton-proton chain also releases energy in the form of a neutrino.

This seems to go beyond the typical "mass and energy can be interconverted", since it appears that we're releasing energy (gamma radiation) **and** the mass is increasing. How is this not a violation of conservation of mass/energy?

Expected Answer: The extra energy comes from the binding energy of the deuterium.

107. (3.00 pts)
In addition to checking whether enough energy is theoretically available, we also need to assess whether these nuclear reactions can physically take place.

Consider an "ideal gas" of protons ($Z = 1$) in the core of the Sun. In order for two protons to undergo a nuclear reaction, let's assume that their centers must be separated by two proton radii. At what temperature, in Kelvin, does the core need to be in order for the average kinetic energy of the protons to be high enough to overcome the Coulombic energy barrier? Show your work.

Hint: $\langle E \rangle = \frac{3}{2} k_B T$

Expected Answer: 6.6×10^9 Kelvin

Quantum tunneling. Detailed calculations of the Sun's interior show that the temperature at its core is likely only 1.1×10^7 Kelvin, which is significantly lower than your answer to the previous question. Since the molecules in an ideal gas have a range of energies, some of the protons will have energies considerably higher than the average, as given by the Boltzmann distribution. However, at a temperature of 1.1×10^7 Kelvin, essentially none of the protons will have energies high enough (here's a Wolfram Alpha link (https://www.wolframalpha.com/input/?i=1-erf%28%281.8e7%29%2F%28sqrt%282%29*4.1e5%29%29-%28sqrt%282%2Fpi%29*%281.8e7%2F4.1e5%29*exp%28%28-%281.8e7%29%5E2%29%2F%282*%284.1e5%29%5E2%29%29%29)). Luckily, quantum tunneling comes to the rescue.

108. (3.00 pts)
One of the most famous postulates of quantum mechanics is the Heisenberg Uncertainty Principle, which states that the product of the uncertainties in position and momentum (Δx and Δp_x, respectively) must be greater than $\hbar/2$. Qualitatively, explain how this could lead to nuclear fusion, even if the energies of the protons isn't high enough.

Expected Answer: Δx is so large that even if the protons have too little energy, the proton might find itself within the potential well of another proton.

109. (3.00 pts)
Suppose that a proton is faced with a barrier of "height" 1.4×10^{-13} Joules and "length" 10^{-13} meters between itself and the potential well of another proton. Estimate the probability the proton is able to tunnel through the barrier. (An order of magnitude approximation is perfectly fine, and feel free to explain your line of reasoning to help get more partial credit.)

Expected Answer: Answers within a couple of orders of magnitude of 10^{-20} will receive full credit. Partial credit for responses that make a significant attempt at answering the question, regardless of correctness.

110. (3.00 pts)
The probability you came up with is still small, but since there are so many atoms in the Sun, fusion is able to occur at a reasonable rate! As MinutePhysics puts it, "it's like finding a needle in a haystack, but the haystack is the Sun".

Let's go a little further, just for fun. In the previous question, we examined the case of two protons colliding, but that isn't always the case: one of the steps in the proton-proton chain involves a deuterium atom and a proton fusing. Let the probability that the deuterium atom tunnels into the proton's potential well be P_1, and the probability from the previous question be P_2. Estimate P_1/P_2, and state the assumptions you make.

Expected Answer: Answers will vary significantly depending on the assumptions competitors make. Typical answers will be around 10^{-1} or 10^{-2}. Full credit will be given for qualitative explanations, even if there is no numerical answer at the end (or if that number is wildly incorrect).

Gravitational waves. Gravitational waves (GWs) are a key prediction of general relativity that have recently emerged as a valuable detection scheme for some of the most energetic phenomena in the universe.

111. (3.00 pts)
In general relativity, mass-energy distributions (such as stars, black holes, etc.) are characterized by a quadrupole moment tensor. Broadly speaking, what must this tensor look like/what property must it exhibit to allow the production of GWs? Your answer should be a single word or short phrase.

Expected Answer: Time dependence, 3 pts all or nothing

112. (3.00 pts)
Let us now estimate the gravitational wave production in the Hulse-Taylor binary system. Assume both components of the binary were identical neutron stars of mass 1.4 solar masses and their orbital period is 8 hours. Modeling the orbits as circular, estimate to order of magnitude the radiated power of gravitational waves from this system in W.

Suppose that your answer is in the form 10^B, where B is an integer. Enter this integer in the blank below.

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113. (3.00 pts) Using your answer to the previous question, compute the approximate orbital decay of this system. Give your answer in meters/year.

Expected Answer: 3.5 m/yr, 3 pts for correct answer. Award partial credit for incorrect answer but approach involving conservation of energy (Sahil can look at this if needed).

114. (3.00 pts)
Another useful parameter to know is the orbital frequency of the system. Indeed, this eventually translates into the characteristic "chirp" we hear from GW spectrograms. Compute the orbital frequency of the Hulse-Taylor binary in μHz.

Expected Answer: 34.72 microHertz. Full credit for answer, partial credit for incorrect answer but process involving Kepler's 3rd law, P = 1/T, or something related.

115. (3.00 pts)
Unfortunately, given the aforementioned parameters, it is unlikely that we will be able to detect the GW output from this system. Explain why. If our detector sensitivities remain the same, is it possible in the (perhaps distant) future to detect GWs from this binary? Explain why or why not.

Expected Answer: Strain is below current sensitivity of LIGO (1 pt), frequency also very low (sub-Hz). Yes, the strain is expected to increase to a detectable level as the neutron stars inspiral (2 pts).

Gravitational lensing. Gravitational lensing is a fascinating phenomenon that both stands the tests of general relativity and allows us to study very distant galaxies. In this problem, we take a closer look into how gravitational lensing really works.

116. (3.00 pts)
Sometimes, gravitational lenses smear out the images of a distant galaxy into a ring-like shape (the so-called "Einstein ring"), whereas sometimes discrete images are produced. What conditions on the gravitational lens and the background source give rise to the difference?

Expected Answer: Pointlike background sources like quasars are lensed as discrete images (1.5 pts) whereas extended background sources like galaxies and galaxy clusters can be lensed as smeared-out rings (1.5 pts). It is important to note that multiple images/rings will only be formed for lenses with sufficiently strong gravitational potentials (optional). A lot of teams are saying that the source and lens must be aligned, which is true, but is not what the question is asking.

117. (3.00 pts)
Piece together what you have found to describe the relationship between deflection (between the image and the source) and the gradient of the gravitational potential of the lens for the images.

Expected Answer: They are directly proportional to one another. 3 pts, all or nothing.

118. (3.00 pts)
It turns out that the relationship you found in the previous question is a direct consequence of the fact that images only form at specific points on the Fermat light travel-time surface. Which types of points are these?

Expected Answer: Critical points (3 pts), i.e. saddle points, maxima, and minima (optional to state what critical pts mean)

119. (3.00 pts)
We often model the gravitational lens as a singular isothermal sphere, where the density profile adopts the form $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$ and the rotation curve is flat. Here, σ is the velocity dispersion and r is the radial coordinate. Using your answer to the previous question, what types of points on the Fermat travel-time surface do we *see*? Why? You can assume the source and lens are approximately aligned.

Expected Answer: Saddles and minima (1 pt); maxima are infinitely demagnified on passing through the center of the lens (specifically, the travel-time surface has a high amount of curvature near the center due to the central peak in the density distribution) (2 pts).

M87*. In this problem, we'll take a closer look at one of your DSOs, M87, and the supermassive black hole at its center, known as M87*.

120. (3.00 pts)
In the now-famous images taken of the supermassive black hole in M87 (M87*) by the Event Horizon Telescope (EHT) (see Image 20), we notice a bright ring surrounding the black hole. Briefly explain what phenomenon best explains the observed position and brightness of this feature.

Expected Answer: Gravitational lensing (2 pts) of radiation from the accretion disk (1 pt)

121. (3.00 pts) Interestingly, we can notice that the bottom of the ring is brighter than the top. Why is this so?

Expected Answer: Doppler beaming (2 pts, mention Doppler effect) due to rotation of the accretion flow (1 pt, should mention something about this rotation)

122. (3.00 pts) What gave EHT the ability to resolve a region so close around M87*?

Expected Answer: Interferometry (2 pt) increased the telescope's effective diameter and thus angular resolution (1 pt for explanation of how interferometry improves angular resolution)

123. (3.00 pts) Now let's take a closer look at the EHT image. Black holes are inherently "black," so where is the bright ring of light actually coming from?

Expected Answer: Accretion disk or photon sphere are acceptable (3 pts)

124. (3.00 pts) It turns out the photon sphere, which is the innermost radius for photons to have (unstable) circular orbits actually lies within the bright ring! Briefly explain why this is.

Expected Answer: The effect is due to gravitational lensing (1 pt). Using the Scwarzschild metric, the ``photon radius'' we observe is dilated relative to its actual value (2 pts)

125. (3.00 pts)
Using the Schwarzschild metric, we can derive that the observed radius of the photon sphere is dilated by a factor of $\sqrt{3}$. Why would slight deviations from this value emerge for M87*?

Expected Answer: The SMBH is rotating, so the Kerr metric (3 pts) is more appropriate, introducing some corrections.